Improving Susceptibility Mapping of Veins Using a K-space Iterative Approach

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Introduction: Mapping susceptibility from field perturbation data is a difficult inverse problem. With some MR specific simplifications, magnetic susceptibility can be calculated by using a fast Fourier transform approach and this has been widely used in many recent works [1-3]. Briefly, the expression for the object's susceptibility distribution, in terms of the phase data can be written as: $\chi(r)=FT^{-1}\{3(k_x^2+k_y^2+k_z^2)/(k_x^2+k_y^2-2k_z^2)\cdot FT[-\varphi(r)/\gamma B_0 TE]\}= FT^{-1}\{g^{-1}(k)\cdot FT[-\varphi(r)/\gamma B_0 TE]\}$ (Eq.1) where, $\varphi(r)$ is the phase distribution, TE is the echo time, γ is the gyromagnetic ratio for hydrogen protons, B_0 is the imaging field strength, and k_x, k_y, k_z are coordinates in k-space. Clearly the analytic inverse filter is not mathematically defined at the points where $k_x^2+k_y^2=2k_z^2$, which poses a problem in using this inverse filter to reconstruct the susceptibility map (SM), $\chi(r)$. To overcome the singularities in the inverse filter and to produce improved accuracy for susceptibility mapping, we propose a novel iterative method which is akin to Partial Fourier [4] reconstruction where we iteratively replace k-space values (in $\chi(k)$) near the singularities to obtain an artefact free susceptibility map. Values used for substitution are estimated using structural information from the masked version of $\chi(r)$.

Materials and methods: The iterative algorithm steps are discussed below and are illustrated in Figure 1. Step-1: An initial estimate of the susceptibility map, $\chi_{i=1}(r)$, is obtained by applying a regularized version of the inverse filter, $g_{reg}^{-1}(k)$ [5], in Eq. 1. The subscript "i" denotes the susceptibility map estimate after the i^{th} iteration step (i=1 for the first iteration). Step-2: Geometry of structures of interest .i.e. venous vessels, are extracted from $\chi_{i=1}(r)$, through a binary mask, to give $\chi_{structures}(r)$ (part (b) in Figure. 1). The mask can be generated from thresholding the susceptibility map estimate, $\chi_i(r)$ itself (or from the magnitude images). Step-3: $\chi_{structures}(k)$ is obtained by Fourier Transformation of $\chi_{structures}(r)$ (part (c) in Figure. 1). Step-4: Extract the k-space values along the cone of singularity and its neighbourhood (part (d) in Figure. 1). This extracted k-space data is denoted by $\chi_{str,\,cone}(k)$. Step-5: Merge data from $\chi_{str,\,cone}(k)$ and $\chi_{i=1}(k)$ (part (e) in Figure. 1). The merged data is denoted by $\chi'_{merged}(k)$ (part (f) in Figure.1). Step-6: Inverse Fourier transformation of $\chi'_{merged}(k)$ gives the improved susceptibility map, $\chi_{i+1}(r)$. Step-7: $\chi_i(r)$ in Step-1 is replaced by $\chi_{i+1}(r)$ and the algorithm is repeated until $\sqrt{\Sigma\left[(\chi_i(r)-\chi_{i+1}(r))^2\right]}/N<\varepsilon$, where N is the number of voxels in $\chi_i(r)$ and ε is the tolerance value determined based on the range of amplitudes within the susceptibility map. To evaluate the efficiency of the algorithm, simulations were performed using standard geometries of cylinders; phase images consisting one diameter 32-pixel cylinder were simulated by $\Delta\chi$ of 0.45 ppm at a Bo value of 3T.

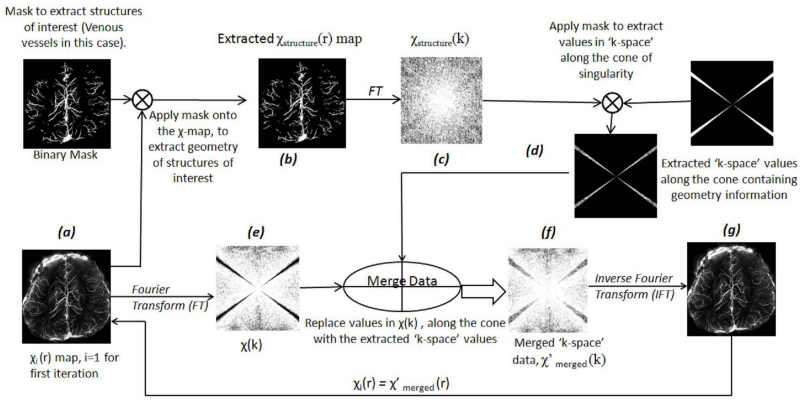

Figure 1: Illustration of the iterative susceptibility map reconstruction algorithm to obtain artefact free susceptibility maps.

Results: In Figure 2.b, the streak artefacts caused by ill-posed problem in the inverse filter are very obvious inside the susceptibility map. As can be seen from Figure 2.c to e, streak artefacts are significantly reduced by using this iterative approach. The removed artefacts are clearly shown in Figure 2.f using a subtraction image. This k-space iterative approach also helps to improve the accuracy of susceptibility values. In general, the susceptibility values in the SM are under-estimated due to regularization and the artefacts. The susceptibility values of Figure 2.b, c d and e are 0.416±0.04ppm, 0.450±0.05ppm, 0.450±0.05ppm and 0.457±0.07ppm. The value from Figure 2e is the convergent susceptibility value. Figure 3 presents the comparison of SM before (Figure 3.a) and after (Figure 3.b) using iterative approach from 0.5mm isotropic resolution data with echo times of 11.6ms. The mean of susceptibility values inside a vessel pointed by a the left arrow in Figure 3.a increased from 0.310±0.059ppm to 0.357±0.066ppm whereas the absolute value of streak artefacts pointed by the right arrow is decreased from 0.092±0.039ppm to 0.060±0.035ppm before and after using iterative approach respectively.

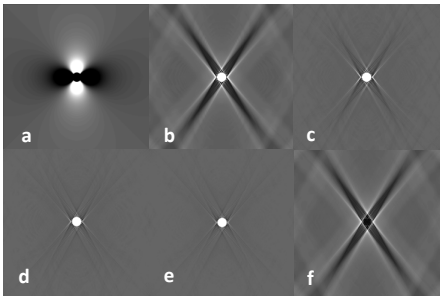

Figure 2: a) Phase images consisting one diameter 32 pixels cylinder were simulated by $\Delta\chi$ of 0.45 ppm at a Bo value of 3T. b) The original SM without using the iterative approach. c) to e) SMs from applying iterative process once to three times. f) The subtraction image, i.e. subtraction image b) from d), which shows how much streak artefacts have been removed by iterative approach. Figures 2.b to f are set at the same display window levels for proper visual comparison.

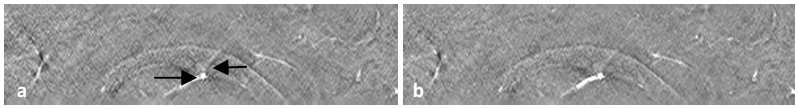

Figure 3: Comparison of SM before a) and after b) using iterative approach from 0.5mm isotropic resolution data with echo times of 11.6ms.

Discussion and Conclusions: The unique k-space iteration/image processing approach proposed here dramatically reduced reconstruction streak artefacts caused by an ill-posed problem of inverse filter and simultaneously improved the accuracy of susceptibility quantification. Compared to the simple threshold based regularization methods [5, 6], this approach improves the accuracy for susceptibility mapping. The approach typically needs less than 10 iterations for the algorithm to converge, compared to the time consuming multiple angle approach [7] and yet provides decent quantitative accuracy. A limitation of this method is the requirement of the vessel geometry information to produce a vessel map for k-space substitution. The better vessel map we obtain; the better is the accuracy we could achieve for susceptibility mapping. Nonetheless, this approach of using geometry information to overcome the illposedness of inverse problem can be applied not just for veins but to other structures as well (e.g. basal ganglia structures). In summary we present here a novel method to remove major artefacts caused by the singularities in the inverse filter to produce an artefact free susceptibility map with improved accuracy. This method could potentially be used for quantitative *in vivo* venous oxygen saturation measurement using SWI data.

References: [1]Salomir et al. Concepts Magn Reson Part B (2003) 198: 26-34. [2]Marques et al. Concepts Magn Reson Part B (2005) 25: 65-78. [3]Koch et al. Phys Med Biol (2006) 51(24): 6381-6402. [4] Xu et al. J Magn Reson Imaging (2001) 14: 628-635. [5] Haacke et al. J Magn Reson Imaging (2010) 32:663-676. [6] Shmueli et al. Magn Reson Med (2009) 62:1510–1522. [7] Liu et al. Magn Reson Med (2009) 61(1):196-204.