In vivo whole brain susceptibility mapping using compressed sensing

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Introduction Quantitative susceptibility mapping potentially allows the in vivo tissue composition to be quantitatively assessed. However, susceptibility mapping from phase measurement is an ill-conditioned process that leads to streaking artifacts in the calculated susceptibility map. Various approaches have been proposed to obtain an artifact free susceptibility map. We here show that the problem of susceptibility mapping can be formulated as a compressed sensing image reconstruction problem. We set out to determine how accurately missing k-space data can be estimated with compressed sensing.

Theory The Fourier transform of susceptibility $x_k(\mathbf{k})$ and that of the phase measurement $\theta_k(\mathbf{k})$ have the following relationship:

$$\theta_k(\mathbf{k}) = x_k(\mathbf{k}) \cdot (1/3 - k_z^2/k^2) = x_k(\mathbf{k}) \cdot D_k(\mathbf{k}) \quad [1]$$

where $D_k(\mathbf{k})$ is the k-space filter that is undefined at $k_z^2/k^2 = 1/3$ that is also known as the conical surface. Due to the vanishing values of the filter at and around the conical surface where $k_z^2/k^2 = 1/3$, the inverse problem from the phase measurement to the susceptibility is an ill-conditioned problem. A straightforward approach to avoid the divide-by-zero problem and reduce the streaking artifacts is to perform a thresholded division in regions where $D_k(\mathbf{k})$ is small as suggested in (1). Specifically, a new k-space filter $D_k'(\mathbf{k})$ is used in the inversion:

$$D_k'(\mathbf{k}) = D_k(\mathbf{k}) \text{ when } D_k(\mathbf{k}) > t; \quad D_k'(\mathbf{k}) = t \text{ when } D_k(\mathbf{k}) \leq t \quad [2]$$

Intuitively, limitation of the thresholded inversion lies in the inaccurate estimation in k-space regions where threshold is made.

Since the severely ill-conditioned k-space regions are the regions whose corresponding $D_k(\mathbf{k})$ values are vanishingly small. In the proposed compressed sensing (CS) based susceptibility estimation, we treat the ill-conditioned k-space regions as missing data and use compressed sensing to retrieve them. The extent to which the k-space regions need to be estimated is determined by a threshold value in $D_k(\mathbf{k})$. The overall susceptibility estimation is a two step process: firstly, a partial k-space estimate is obtained by performing a direct inversion using Eq. [2] up to a set threshold level; in the second step, the void in the resulting k-space dataset is compensated using compressed sensing. Mathematically, the proposed approach can be written as:

$$x_k'(\mathbf{k}) = \theta_k(\mathbf{k}) \cdot D_k'(\mathbf{k})^{-1} \quad [3]$$

where $D_k'(\mathbf{k})$ is the same as that in Eq.[2] and $x_k'(\mathbf{k})$ is the resulting susceptibility estimate using thresholded inversion. The compressed sensing compensated estimate of the susceptibility x' is obtained as (2):

$$x' = \min_x ||x_k' \cdot \mathbf{h} - \text{diag}(\mathbf{h})\, \mathrm{W}x||_2 + \alpha||\Phi x||_1 \quad [4]$$

where **h** is a binary mask determined by the threshold level *t*, W is a Fourier matrix and diag(**h**) has **h** on the diagonal and zero elsewhere; Φ is an appropriate sparse transform and α is the corresponding weighting coefficient.

Method In vivo brain imaging of a healthy adult volunteer was performed on a 3T GE MR750 scanner equipped with an 8-channel head coil. A standard flow-compensated 3D SPGR sequence was used with the following parameters: FOV = 256×256×180 mm^3, matrix = 256×256×180, TE/TR = 40/50 ms, flip angle = 20º. Phase images were first extracted from each coil and individually performed with phase unwrapping and background phase removal using the method proposed in (3). The resulting phase maps from each coil were then averaged to obtain the final phase map, which are then used to calculate the susceptibility maps.

Results and discussion A saggital plane of the calculated susceptibility maps using direct threshold method and with CS estimation are shown in Figure 1 at various threshold levels. It is seen that the susceptibility maps obtained using CS compensated method lead to much lower level of streaking artifacts at all the threshold level tested. In the case, CS based method lead to streaking artifact free estimate with a threshold level of 0.0625. In addition, there is a suspected susceptibility source (pointed by arrow) in the direct threshold estimates but seen to be non-existent in the CS compensated estimates. The latter is consistent with the observation in magnitude image (image not shown).

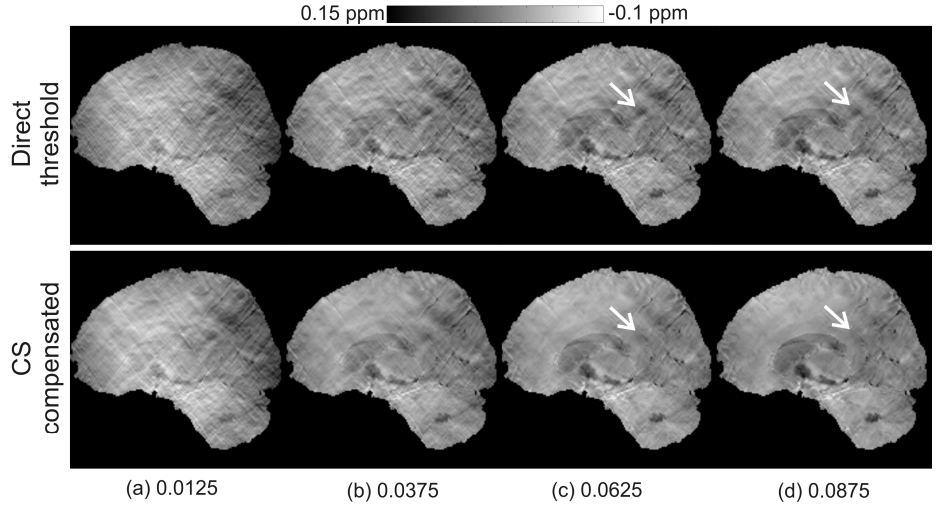

Figure 1 Saggital slice of susceptibility calculated at various threshold levels.

Conclusion We presented a novel CS based method for susceptibility mapping, in which the ill-condioned k-space regions are estimated using CS. Susceptibility estimates with much lower level of streaking artifacts are received comparing to those obtained using direct threshold method.

Reference (1) Shumeli, et al., MRM 2009. (2) Lustig, et al., MRM 2007. (3) Schweser, et al., Neuroimage 2010.