Quantitative Susceptibility Imaging using L1 regularized reConstruction with Sparsity Promoting Transformation: SILC

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Introduction:
Phase image has been shown to provide superb tissue contrast at high imaging field, and reveals anatomical details that cannot be easily identified in magnitude images. Reconstruction of the susceptibility distribution from phase images provides a quantitative measure of tissue properties, but faces many difficulties, including zero points in the convolution kernel. Methods have been proposed to solve the problem (1-2). Here we develop a novel method of reconstructing susceptibility distribution from phase images, which borrows ideas from compressed sensing.

Theory:
The relationships between susceptibility distribution $\chi(\vec{r})$, field perturbation $\Delta B_z(\vec{r})$ and phase $\psi(\vec{r})$ are given by:

$$\Delta\hat{B}_z(\vec{k}) = B_0 \cdot \hat{\chi}(\vec{k}) \cdot C(\vec{k}) \ , \quad C(\vec{k}) = (\frac{1}{3} - \frac{k_z^2}{|\vec{k}|^2}) \qquad \text{[Eq. 1]}$$

$$\psi(\vec{r}) = -\gamma \cdot \Delta B_z(\vec{r}) \cdot TE \qquad \text{[Eq. 2]}$$

where B_0 is the main field strength, $\Delta\hat{B}_z(\vec{k})$ is the Fourier transformation of $\Delta B_z(\vec{r})$, $\hat{\chi}(\vec{k})$ is the Fourier transformation of $\chi(\vec{r})$, $\vec{k}$ is the coordinate in the Fourier space, kz is the z component, γ is the gyromagnetic ratio and TE is the echo time. Reconstruction of $\chi(\vec{r})$ from $\Delta B_z(\vec{r})$ is ill-posed as the convolution kernel $C(\vec{k})$ has zero-points. These zero-points can be viewed as un-sampled data points of $\chi(\vec{r})$ in the Fourier domain. Compressed sensing with L1 norm regularization term was found to be able to reconstruct images with good fidelity from under-sampled data. We therefore propose to formulate the above reconstruction method as below:

$$\chi = \arg\min_X (1/(2\rho)\,|\, A \cdot X - b\,|_2 + |\, W \cdot X\,|_1) \qquad \text{[Eq. 3]}$$

where $A = ifft \cdot C(\vec{k}) \cdot fft$, b is the relative field perturbation $\Delta B_z/B_0$, W is a sparse transformation, e.g., wavelet transformation, ρ is a tuneable parameter which controls the balance between the data consistency and solution sparsity in the transformed domain given by W.

Methods:
A tube of susceptibility value of 0.02ppm with radius of 8mm perpendicular to the main field was simulated for a field strength of 3T and a TE of 30ms. Equations 1-2 were used to generate the phase image, and different levels of random noise were added to the phase image with standard deviation of 0.001, 0.005, 0.01 and 0.05 rad respectively. The proposed SILC method (Eq. 3) was used to reconstruct the susceptibility distribution. Wavelet transformation was used as the sparse transformation (Debauchies with filter length of 6). Different values of ρ were tested, which included 1e-6, 1e-5, 5e-5, 1e-4, 1e-3 and 1e-2. As a comparison, a direct reconstruction method based on truncation of the kernel $C(k)$ was also implemented (referred to as KM method below) (2). Different truncation levels (α) were tested, including 0.05, 0.1, 0.15, 0.20, 0.25 and 0.30. The performance of the methods with different parameters was also tested on in-vivo human brain data acquired using a flow-compensated SPGR sequence at 3T with the following parameters: TR/TE = 46/30ms, Matrix = 192x256, In-plane resolution = 0.94mm, thickness = 1mm and number of slices = 110. The phase image was high-pass filtered first to remove the background field variation before applying the reconstruction algorithms.

Results: Simulation analysis showed that there were residual streaking artifacts in the KM reconstructed susceptibility image while SILC nearly perfectly reconstructed the susceptibility image. For the KM method, the mean estimated susceptibility value inside the tube was found to depend on the truncation level and largely underestimated the true value (0.02ppm) (Fig. 1a). In contrast, for the SILC method, the mean estimated susceptibility value closely approached the true value for a range of values of ρ (Fig. 1b).

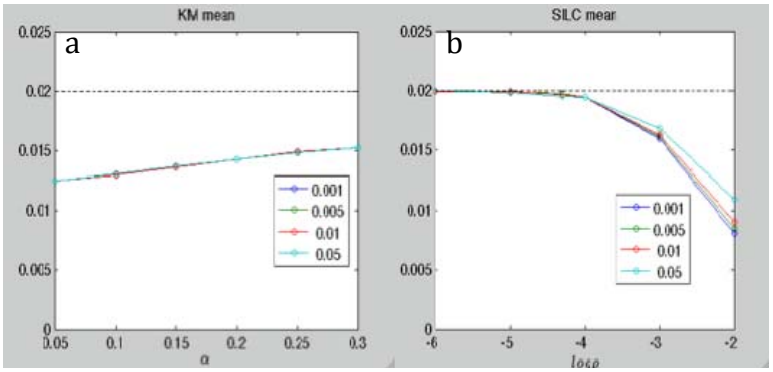

Fig. 1 shows the mean of estimated susceptibility value inside the tube for KM method (a), and SILC method (b) with different parameters. The horizontal gray dotted line indicates the true value

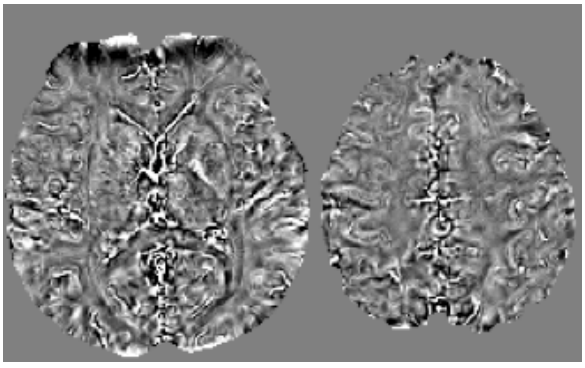

Fig. 2 shows SILC reconstructed susceptibility image with ρ=1e-4

The SILC method successfully reconstructed susceptibility distributions from the in-vivo human brain data, and the contrast matches known distribution of susceptibility in the brain (Fig. 2). The susceptibility image reconstructed using KM method showed lower resolution, possibly due to modification of the kernel; and the values were lower than that in SILC reconstructed image, consistent with simulation results.

Discussion: We have proposed a novel method (SILC) for reconstructing susceptibility image from field map using L1 regularization with sparsity-promoting transformation. Simulations showed high fidelity of the reconstructed image using SILC to the gold truth for a range of the data consistency penalty parameter ρ. This quantitative susceptibility imaging technique has many potential applications, including measurement of blood oxygenation level in the vessels, quantification of iron loading in the brain, definition of arterial input function in dynamic susceptibility contrast imaging.

References: *1.* Wharton S. 2010. NeuroImage. 53: 512-25. *2.* Haacke E. 2010. JMRI. 32:663-76. *3.* Shmueli K. 2010. MRM. 62:1510-22.

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