G-factor as Regularization Parameter in Regularized SENSE Reconstruction

H. Omer[1], and R. Dickinson[2]
[1]Imperial College London, London, London, United Kingdom, [2]Imperial College London

Introduction: Parallel MRI is a technique to reduce the scan time for MRI image acquisition. The acceleration is achieved by reducing the number of phase encode steps during the acquisition of k-space. The inverse Fourier Transform of this reduced k-space produces aliased images. SENSE reconstruction [1] uses the sensitivity maps of the receiver coils to allocate the aliased signal components at their correct pixel locations thus reconstructing a full Field-of-View (FOV) image. A limitation of standard SENSE (especially for higher acceleration factors) is the noise amplification and the residual aliasing artefacts which badly affect the quality of the reconstructed image. The geometry factor (g-factor) [1] represents the noise amplification during the process of image reconstruction and it varies from pixel to pixel. The g-factor term arises as a result of the coil sensitivities being too similar and this ill-conditions the matrix inversion at the heart of parallel imaging. [2]

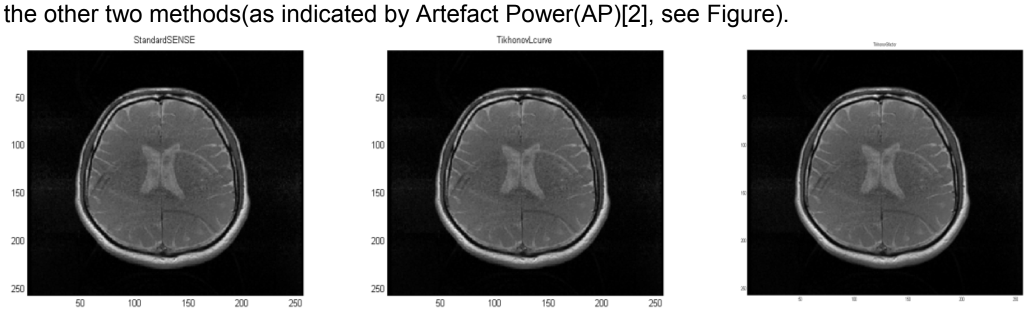
Methodology: Regularization is a method to use prior knowledge to improve the quality of the reconstructed image. A low resolution full FOV image of the anatomy of interest (acquired as a quick scan) can be used as a prior information in Tikhonov Regularization [3]. The following equation shows Tikhonov regularized solution:

$$M_{reg} = \arg\min\{\| CM - I \|_2 + \lambda^2 \| A(M - M_r) \|_2\} \quad (1)$$

Here ‘λ’ is a regularization parameter, ‘A’ is a function, in our case identity matrix. ‘M_r’ denotes the prior information, ‘M’ is the non-regularized solution image and $\|---\|_2$ denotes the L-2 norm. The first term in the above equation represents the deviation of the observed aliased image (I*)* from the model observation, giving the Model Error. The model observation is a folded version of the initial solution image ‘M’. The second term, defined as Prior Error, shows the deviation of the solution image ‘M’ from the prior knowledge (M_r).The regularization parameter (λ) determines the relative weights with which these two estimates of error combine to form a cost function. A good estimation of the regularization parameter is important to have a good regularized reconstruction. Recently an L-curve method [3] has been proposed to choose an optimal regularization parameter. Here the Model Error is plotted vs. Prior Error for a range of λ values (normally between the singular values of the encoding matrix). The optimal regularization parameter is defined as the one which strives to minimize the balance between the two terms (at the elbow of the curve). This is computationally quite expensive because the process needs to be repeated for all pixel values.

We suggest the use of g-factor itself as a regularization parameter rather than choosing it through a complex computation. A g-factor value of 1 indicates no noise amplification at that pixel location and higher values show greater noise amplification. It is quite logical to use g-factor as regularization parameter because if the g-factor is higher at a pixel location, we would like to depend more on the prior image (and would like to give greater weight to the second term in equation 1) as higher noise makes the model (first term in equation 1) more unreliable. If the g-factor value is closer to 1 at a pixel location, meaning less noise amplification, it is better to depend more on the first term to have a good quality reconstruction. This is computationally quite efficient because g-factor can be calculated very quickly as a standard practice in SENSE reconstruction.

Results: The method was tested on the phantom as well as human brain images obtained by 1.5 Tesla GE scanner at St. Mary’s Hospital London with an eight channel head coil and a Gradient Echo sequence with the following parameters: TE=10 m sec, TR=500 msec, FOV=20 cm, Bandwidth=31.25 KHz, Slice Thickness=3 mm, Flip Angle=$90^{o,}$ Matrix Size=256x256. The images below show the reconstruction results by Standard SENSE, L-Curve-Tikhonov Regularization and g-factor Tikhonov Regularization for an acceleration factor of 4. The results show that the proposed method provides better reconstruction as compared to the other two methods(as indicated by Artefact Power(AP)[2], see Figure).



a b c

Figure 1: Reconstructed images of human brain data for an acceleration factor of 4: a) Standard SENSE reconstruction, AP=0.0120, b) L-curve Tikhonov regularization, AP=0.0113, c) g-factor Tikhonov regularization, AP=0.0097

Conclusion: A method based on the use of g-factor as a regularization parameter in the Tikhonov regularized reconstruction is proposed. The results show a considerable improvement in the reconstructed images as compared to contemporary methods. It has been shown that the g-factor map effectively acts as a spatially adaptive regularization parameter providing very good reconstruction results at much less computational cost.

References:
[1] Preussmann K P et al, SENSE:sensitivity encoding for fast MRI, Magnetic Resonance in Medicine, 1999, 42:952-962
[2] Omer H, Dickinson R, ‘A Graphical Generalized Implementation of SENSE reconstruction using Matlab’ Concepts in Magnetic Resonance Part A, 2010, 36,3, 178-186, John Wiley & Sons
[3] Lin FH, et al. 2004. Parallel imaging reconstruction using automatic Regularization. Magn Reson Med 51:559–567.