# The Accuracy of Noise Covariance Estimation and Its Relationship with Signal-to-noise Ratio in Parallel Magnetic Resonance Imaging

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**Introduction:** Parallel magnetic resonance imaging (pMRI) techniques are widely used in abdominal and thoracic imaging applications. The k-space raw-data is down-sampled to save scan time, and a set of linear equations is solved to reconstruct the image. As noise from different phased-array coil channels is partially correlated, the least-square (LS) solution is sub-optimal. Hence, image based pMRI reconstruction algorithms (SENSE or its variants) use the best linear unbiased estimation (BLUE) which uses the noise covariance matrix to de-correlate the noise [1]. BLUE degenerates to the LS when noise from different channels is uncorrelated, which is rarely the case in practice. Mathematically, the signal-to-noise-ratio (SNR) of images reconstructed by BLUE is better or equal to the SNR of LS, depending on the accuracy of the noise covariance matrix estimate. There are two sources of errors in covariance matrix estimation in cardiac imaging: first, the noise sample size which influences the random fluctuation of the noise covariance matrix estimation; second, in the case of abdominal or thoracic imaging, respiratory motion which changes the coil geometry and loading. To date, it has not been determined how these two types of errors affect the SNR of pMRI. In this study, we sought to quantify the relationship between the two types of errors, and to propose a guideline for accurate covariance matrix estimation.

**Methods:** We acquired real-time cardiac cine images on a 1.5T MR scanner (MAGNETOM Avanto, Siemens Healthcare, Germany) with fully-sampled k-space in one short-axis view in seven healthy volunteers during inspiration. Two noise scans, each containing $4\times10^6$ noise samples were acquired separately during inspiratory and expiratory breath-holds, respectively. Imaging parameters were: 160 × 120 matrix, 8mm thick slice, flip angle=$74^o$, TE/TR = 1.18/2.35 ms, pixel bandwidth= 1645 Hz/pixel, FOV = 400 × 300 $mm^2$. A 12-channel standard body matrix coil was used for data acquisition, and the same experiment was repeated using a 32-channel coil (Rapid MRI, Columbus, OH). The noise covariance matrices estimated from two separate noise scans using variant number of noise samples were compared using the metric proposed by Smith[3]. The difference ($d_{\mathrm{cov}}$) of two covariance matrices $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, is defined as the 2-norm of the vector of logarithms of the generalized eigenvalues $\{\lambda_k\}$. When random fluctuation is the only source of error, $d_{\mathrm{cov}} \sim N^{1/2}$, where $N$ is the number of noise samples included. We evaluated $d_{\mathrm{cov}}$ vs. $N$ to study the effect of the covariance matrix difference induced by the respiratory motion. The k-space data was down-sampled to perform off-line SENSE reconstruction. From each noise scan, a series of noise covariance matrices were estimated by varying the number of noise samples included. Each noise covariance matrix was used to reconstruct an image series with BLUE, and the image SNR gain relative to the LS solution was measured using the recently proposed random matrix theory based method [2]. The SNR gain was plotted vs. $N$ to study the effect of noise sample size.

**Results:** Figure 1 shows the $d_{\mathrm{cov}}$ of three pairs of noise covariance matrices vs. $N$ using both 12-channel and 32-channel coils in one volunteer study. When two noise covariance matrices were estimated from two noise scans both acquired during inspiration or expiration, the differences followed that expected due to random fluctuation. However, the differences of two covariance matrices estimated from inspiration and expiration noise scans deviated significantly from random fluctuation when more than $10^4$ samples were included. Figure 2 shows the final image SNR as a function of covariance matrix estimated using variant number of noise samples. The SNR gains of BLUE increase when more noise samples are included, and saturate when $N > 10^4$. Using 99% of the maximum image SNR gain as the tolerance, the minimum $N$ is summarized in Table I. To gain the SNR advantage of BLUE, $N > 10^4$ is required, and the 32-channel coil requires higher $N$ than 12-channel coil. Respiratory phase did not have a significant impact on the noise covariance matrix estimate. When applying the noise covariance matrix acquired during inspiration to reconstruct images acquired during expiration, or vice versa, the difference of SNR gain was less than 1%.

**Discussion and Conclusion:** The results provide a greater understanding of how the accuracy of the noise covariance matrix estimation affects the SNR of SENSE. Using both 12-channel and 32-channel phased-array coils, we demonstrated that BLUE can improve the image SNR significantly compared to LS solution, and the accuracy of the noise covariance matrix estimation strongly affects the SNR performance. The number of noise samples acquired is a major factor determining covariance matrix accuracy and reconstructed image SNR. The effect of respiration motion was observable but did not affect the image SNR significantly. Our results suggest two guidelines to follow: (1) utilize at least $10^4$ noise samples for noise covariance matrix estimation, (2) only one noise scan should be necessary for an entire patient study as the covariance matrix estimate is unaffected by respiratory phase.

**References**: [1] Pruessmann, KP et al, MRM 42: 952 (1999). [2] Ding Y *et al*, MRM 63: 782 (2010). [3] Smith ST *IEEE TSP*. 53(5):1610 (2005). [4] Sengupta AM and Mitra PP, Phys. Rev. E 60, 3389 (1999).

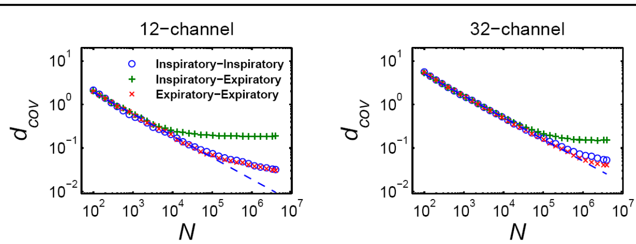


Figure 1: The differences of the noise covariance matrices pairs vary with the number of samples $N$ included in one volunteer study. When the two covariance matrices are estimated from two scans at the different respiratory phases, significant deviation from the random fluctuation ($d_{\mathrm{cov}} \sim N^{1/2}$, indicates by the dashed line) can be observed. .

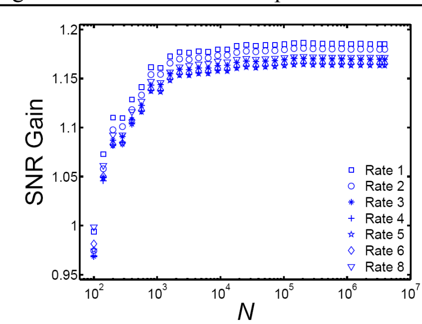


Figure 2: The SNR gain of BLUE over LS solution vs. number of noise samples $N$ included for noise covariance matrix estimation in a 32-channel coil at variant SENSE acceleration rate. The SNR gain plateaus when $N > 10^4$ .

Table I  Minimum Noise Sample Size

| | Median | Minimum | Maximum |
|---|---|---|---|
| 12-channel | 1,600 | 100 | 9,051 |
| 32-channel | 3,394 | 400 | 25,600 |