Rapid Sample Density Estimation for 3D Trajectories

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Introduction: The reconstruction of non-cartesian k-space trajectories often requires the estimation of non-uniform sampling density. Particularly for 3D, this calculation can be computationally expensive. The method proposed in this work combines an iterative algorithm previously presented by Pipe [1] with the optimal kernel design previously presented by Johnson [2]. The combined implementation shows significant time reductions (of up to 100 times) in estimating the densities of 3D center-out trajectories as compared to the approach suggested in [2]. This method is also separable, providing straightforward implementation on parallel computing platforms.

Methods: _Density Estimation_: Each step in the iterative method [1] requires a convolution between the sample density weighting function (W) and a convolution kernel (C) to determine the next set of weights (equation 1).

$$W_{i+1} = \frac{W_i}{W_i \otimes C} \tag{1}$$

Evaluation of this convolution can be implemented directly as shown on the left hand side of equation 2, or through a two step convolution process employing gridding as an intermediate step as shown on the right hand side where 'G' represents the grid (or shah) function.

$$W \otimes C_{direct} \approx (((W \otimes C_{grid}) \cdot G) \otimes C_{grid}) \tag{2}$$

The time savings of the proposed method is due to the gridding step which effectively removes the need for searching through non-uniformly spaced data in order to perform the convolution. In order to maintain the same effect as the direct kernel (C_{direct}) the grid kernel (C_{grid}) is designed in the spatial domain to be

$$c_{grid} = \sqrt{c_{direct}}, \tag{3}$$

where 'c' is the fourier transform of the convolution kernel 'C', as proposed in [1]. _Simulations_: The 3D spiral trajectory [3] shown in figure 1e consists of fully sampled spiral planes rotated at even angles about the k_z axis. The trajectory supports a 64 diameter matrix using 101 planes and 11 archimedean spiral interleaves per plane. Data were simulated from a T_1-weighted, 64^3 image volume. The computational time was measured with similar trajectories using 21, 31, 41 and 51 archimedean spiral interleaves per plane to simulate different levels of local density (eg. around the center of k-space). Density corrected data were reconstructed using the discrete fourier transform. 16 iterations were used in each method. Benchmarks were performed on an Intel 3.2 GHz 8-CPU.

Results: The benchmarks in figure 1a show the run time of the direct method grows precipitously with the density, whereas the grid method scales more linearly. Figures 1c and d were reconstructed using non-parallel grid and direct methods yielding the same RMSE at durations of 70 and 250 seconds respectively.

Conclusion: The proposed grid method is capable of producing sample density estimates comparable to the direct method in reconstruction error. For trajectories with areas of high local densities, such as 3D center-out trajectories, the grid method provides a practical computation time. Parallelization of the grid method yields significant time reductions.

References: [1] Pipe, MRM, 41:179, 1999; [2] Johnson, MRM, 61:439, 2009; [3] Irarrazabal, MRM, 33:656, 1995.

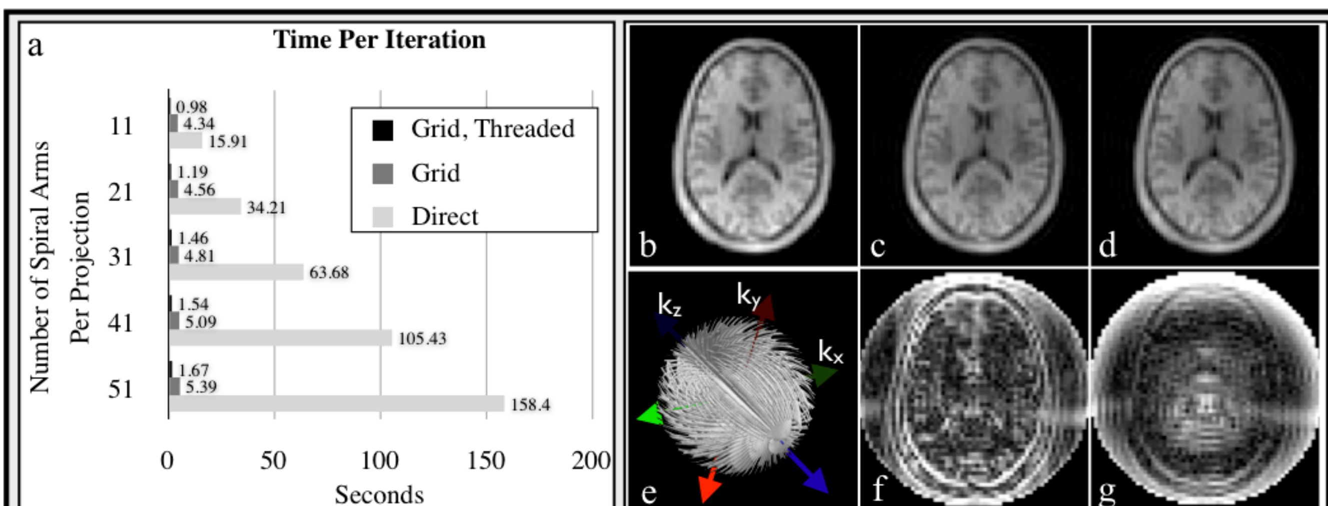

Figure 1: SDC compute time at various sample densities (a), simulation truth image (b), reconstruction using the grid method (c), direct method (d), and the respective difference images (f-g). (e) 3D spiral-projection trajectory used in sims.