Bloch Equation Based Algebraic Reconstruction for MRI using Frequency-Modulated Pulses

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Introduction

Magnetic resonance imaging of ultrashort T_2^* spins generally requires the use of a short radiofrequency pulse delivered with high peak power. In contrast, frequency-modulated (FM) pulses enable uniform spin excitation with a relatively low peak power, because they deliver a flat excitation profile and energy is distributed in time due to sequential excitation of spins by the frequency sweep. However, the spin excitation with an FM pulse induces a quadratic phase distribution that can not be retrieved by application of linear magnetic field gradients (1). The recently described FM-pulse-based ultrashort T_2^* imaging method known as SWIFT (2) overcomes this quadratic phase problem by the correlation method (3). More recently, an algebraic reconstruction method based on an approximation of instantaneous excitation along with the frequency sweep has also been introduced and used to reconstruct SWIFT image (4). However, while these two methods regard spin dynamics as a linear system, the actual spin system has non-linearity in its time evolution that is described by the Bloch equation. Hence, incorporation of the non-linearity into the reconstruction method should improve the image quality. Here, we introduce a Bloch equation based algebraic reconstruction method applied to COncurrent Dephasing and Excitation (CODE) with FM pulse excitation (Fig.1).

Theory

In CODE, k-space sampling is performed in a radial manner, as the orientation of the magnetic field gradient changes in a stepwise manner (5). The experimentally acquired signal vector composed of N complex sampled points, $S(t_i)$, is expressed as

$$S(t_i) = \sum_{k=1}^{M} P(x_k, t_i)\rho(x_k), \qquad i = 1, \cdots, N,$$

where $\rho(x_k)$ is one projection of the spin density on the applied gradient direction, and $P(x_k,t_i)$ describes time evolution of the isochromat at position x_k. $\rho(x_k)$, that is to be obtained, is solved by calculating the inverse of $P(x_k,t_i)$ and then taking the matrix product with $S(t_i)$.

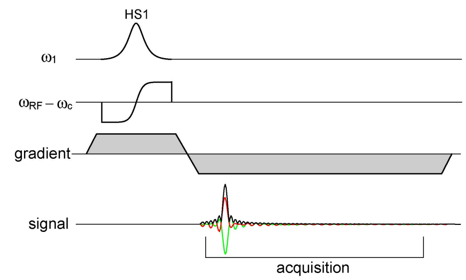

Figure 1. Sequence diagram of CODE.

Method

First, a 1-dimensional (1D) numerical simulation was performed. The signal $S(t_i)$ was generated by numerically integrating the Bloch equation. A hyperbolic secant (HS1) pulse with length T_p = 250 μs and time-bandwidth product of 20 was employed as an excitation FM pulse. The time evolution matrix $P(x_k,t_i)$ was calculated from the analytical solution of the Bloch equation (6). Second, experimental data was acquired on a Varian 16.4 T MRI system using an agar gel phantom. The same HS1 pulse used in the simulation was also employed as an excitation pulse in experiments. Sequence parameters in CODE were TR = 4.1 ms, TE = 400 μs, sw = 80 kHz, FOV = 5×5×5 cm^3 and number of projections = 256000. In addition to the time evolution matrix in the simulation, the $P(x_k,t_i)$ containing T_2^* relaxation effects was generated by performing a numerical Bloch simulation with T_2^* = 10 ms. Each acquired data was reconstructed to one projection by using the $P(x_k,t_i)$s. The resultant projections were transformed to time domain signals with the inverse Fourier transformation (FT). After performing gridding on a Cartesian 3D k-space (7), a 3D image with a matrix size of 400×400×400 was reconstructed with FT. For comparison, reconstruction with the correlation method was performed. All the simulation and image reconstruction were performed with in-house tools programmed in Matlab (Matlab 7.9, R2009b, The Mathworks Inc.).

Results and Discussion

Since the CODE signal is an asymmetric echo due to a much longer acquisition time than T_p (Fig.1), the 1D numerical simulation exhibited severe ringing artifacts in the profile when using FT due to truncation of the signal (Fig.2a). In contrast, the algebraic reconstruction reproduced the spin density profile with very high accuracy. While there was a conspicuous quadratic phase in the FT profile, the algebraic reconstruction completely compensated the quadratic phase (Fig.2b).

Reconstructed images from experimental data are shown in Fig.3. While signal-to-noise ratio (SNR) was comparable regardless of the reconstruction method (Fig.3a-c), sharpness of the edge was improved with the algebraic reconstruction relative to that from the correlation method (Fig.3d,e). Additionally, incorporation of T_2^* relaxation into the reconstruction method further decreased blurring of the edge (Fig3f), because T_2^* decay of signal induces image blurring, especially at higher magnetic fields like 16.4 T used in this study.

This Bloch equation based reconstruction method is not limited to CODE, but is applicable to many types of MRI sequences, including SWIFT. Furthermore, this method has the capability to include physical factors such as T_1 and T_2 relaxation. Although we focused only on T_2^* relaxation effects (i.e. sharpness of the edge) in this study, the method have potential to improve other image qualities such as resolution and contrast, and enhance valuable information in various biomedical applications.

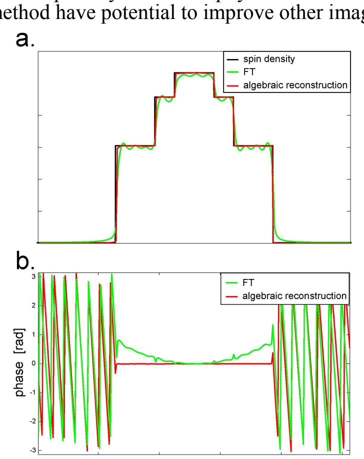

Figure 2. Numerical CODE simulation.

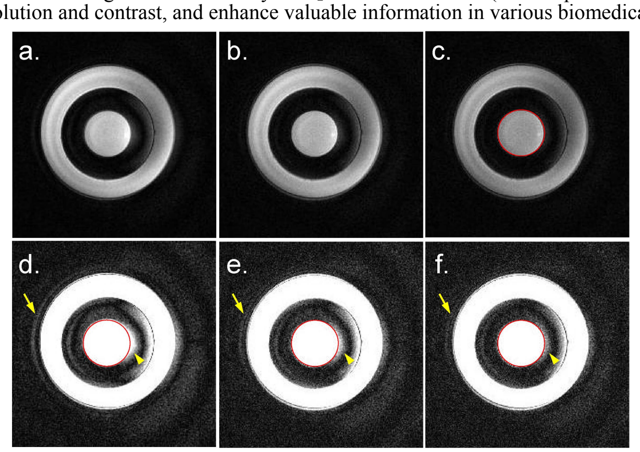

Figure 3. Reconstructed CODE images with the correlation method (a,d), the algebraic method without T_2^* relaxation (b,e), and the algebraic method with T_2^* relaxation (c,f). The window level of the images on the bottom row was clipped in order to enhance the sharpness of the edge. Red circles are the edge of the inner object extracted from the image c.

Acknowledgements

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References

(1) Park JY, et al., *Magn. Reson. Med.* 2006;**55**:848-857. (2) Idiyatullin D, et al., *J. Magn. Reson.* 2006;**181**:342-349. (3) Dadok J, et al., *J. Magn. Reson.* 1974;**13**:243-248. (4) Weiger M, et al., *Magn. Reson. Med.* 2010; in press. (5) Park JY, et al., Proceeding of the 17th annual meeting of ISMRM, 2009, pp.2979. (6) Silver MS, et al., *Phys. Rev. A* 1985;**31**:2753-2755. (7) Jackson JI, et al., *IEEE Trans. Med. Imaging* 1991;**10**:473-478.