

On the Spatiotemporal Bandwidth of Cardiac Motion

M. E. Brummer¹, M. Sanz-Blasco², S. Krishnan³, L. H. Hamilton⁴, S. Ramamurthy³, and D. Moratal⁵

¹Pediatrics and Radiology, Emory University, Atlanta, GA, United States, ²Universitat Politècnica de València, València, Spain, ³Pediatrics, Emory University, Atlanta, GA, United States, ⁴Bioengineering, Georgia Institute of Technology, Atlanta, GA, United States, ⁵Center for Biomaterials and Tissue Engineering, Universitat Politècnica de València, València, Spain

Introduction

In design of sparse sampling strategies determination of the relative importance of high vs. low spatial frequencies remains a controversial issue. Low spatial frequencies contain most signal energy and are often sampled more frequently to increase overall SNR, but for dynamic cardiac imaging we reported recently our observations that high spatial image frequency data are subject to greater change [1]. In this work we take a more theoretical and analytical look at spatiotemporal spectral content of moving objects in MRI, and formulate corollaries for sampling strategies.

Methods

Theoretical Background: Let us observe spatial (x) motion in a time-resolved function $f(x, t)$. An important, commonly encountered and easily modeled, motion element is linear translational motion: $f(x, t) = f(x - x'(t))$. It is well known that the Fourier Transform of an image is modified by translation through a k -proportional phase warp: $F(k, t) = F(k) e^{2\pi i k x'(t)}$. Eq. (1) shows that the change at each spatial frequency k , i.e., the temporal derivative of this function, is related to the value of F , but amplified linearly by the spatial frequency. In other words, the *rate of change* is *linearly proportional to the spatial frequency* (eq. 2), where $g(t)$ represents all constant factors and k -independent changes.

$$\frac{\partial F(k, t)}{\partial t} = 2\pi i k F(k, t) \frac{\partial x'(t)}{\partial t} \quad (1)$$

$$\frac{\partial F(k, t)}{\partial t} / F(k, t) = k g(t) \quad (2)$$

Experimental Design: For experimental confirmation of this finding we analyzed noiseless kt -space data of a cardiac motion phantom introduced in [2]. The phantom simulates various types of motion including translation and concentric contraction. Lines were selected (see Fig. 1) from the image at locations subject to translational motion and to contractile motion following a pattern derived from an actual cardiac MRI scan with 6ms high temporal resolution. Spectra were observed in phantoms generated with various motion amplitudes. 3-D k_x, k_y, k_z spectral density plots were generated after zeroing the dominant static ($k_t = 0$) spectral component for improved visibility. Phantom data were generated on a 128x128-point k -space/image grid, at 72 time samples across the cardiac cycle. Translational motion amplitudes were varied from 10.625 to 42.50 pixels. For contractile motion realistic amplitudes relative to the field of view (FOV) were scaled by factors 0.5 to 3.0.

Results and Discussion

The spectral plots in Figure 2a-c, which represent translational motion with sinusoidal temporal dynamics at amplitudes 21.25, 10.625, and 42.50 pixels respectively, confirm that the temporal (k_t) bandwidth is a linear function of the spatial frequency. Similar k_x -proportional temporal bandwidth is observed with concentric contractile motion, for which spectral plots are shown in Figure 2d-f for mid-, low-, and high-amplitude motion respectively. In both types of motion, the temporal Nyquist frequencies, above which no spectral content is observed, show linear dependence on the motion amplitude for all spatial frequencies. As expected, we did not observe any correlation between object size and temporal Nyquist frequency for translational motion.

These observations are interesting for several reasons. First, these experiments appear to indicate that there actually exists a clearly defined temporal Nyquist frequency for each k_x given certain spatial sampling and motion amplitude. This suggests that it may be possible to determine the required number of time points for certain classes of cardiac images on a known spatial sampling grid size. Secondly, these results imply that since the temporal Nyquist frequency increases with k_x , high spatial frequencies need to be sampled more frequently for faithful image reconstruction. This is contrary to the common practice of oversampling low frequencies for noise gain or motion correction.

Conclusions

Temporal Nyquist sampling rates increase with the spatial frequency for translational and contractile periodic motion, as can be observed experimentally by spectral analysis. This is important for optimization of clinical protocols and for design of novel sparse sampling strategies.

References

1. M Sanz Blasco, et al., *Proc. of the 17th ISMRM*, Honolulu, 2009

2. ME Brummer et al, *Magn Reson Med*, Vol 51(2), pp. 331-342, Feb. 2004.

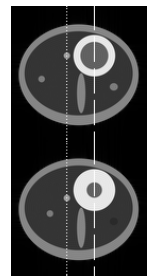


Figure 1: Diastolic (top) and systolic (bottom) phantom frames. Dotted and dashed lines were used for translational and contractile motion analysis,

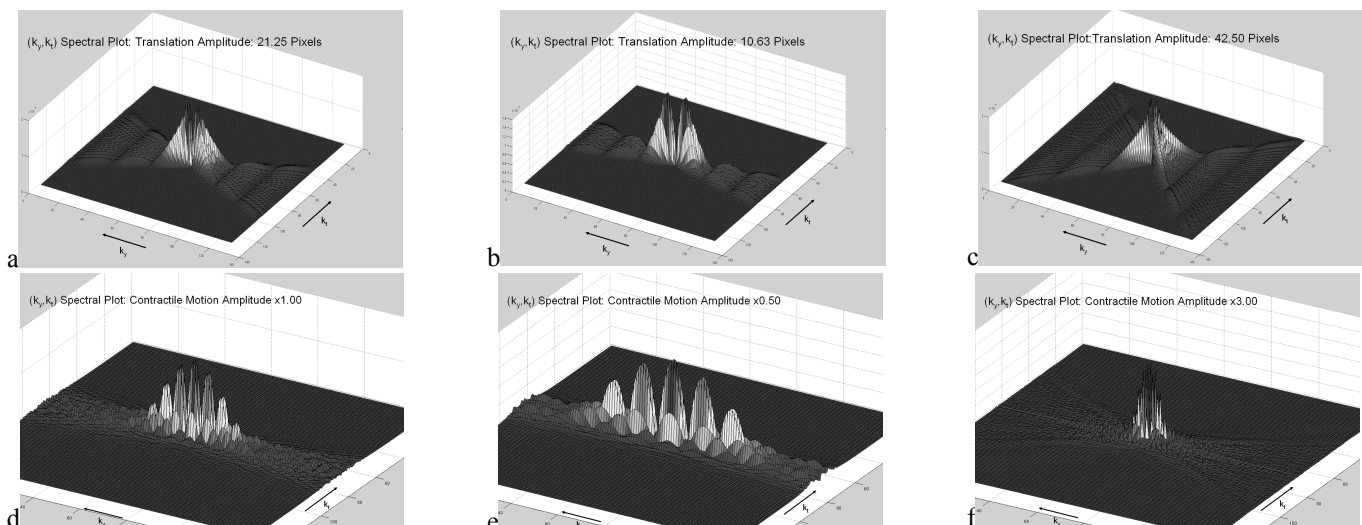


Figure 2: Spatiotemporal spectral plots for phantom image lines at readout locations with translational (a-c) and contractile (d-f) motion at three different amplitudes.

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