

# Local versus Global Low-Rank Promotion in Dynamic MRI Series Reconstruction

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**Introduction:** Several recent works (e.g. [1-3]) have suggested that dynamic MRI series reconstructions can be significantly improved by promoting low-rank (LR) structure in the estimated image series when it is reshaped into Casorati form ( $N \times N \times T$  series  $\rightarrow N^2 \times T$  matrix) [2]. When  $T \ll N^2$ , the rank of the (reshaped) true underlying image may actually be not much less than  $T$ . For such cases, aggressive rank reduction will result in temporal blurring [3] while only modest rank reduction will fail to remove noise. In [4], Bydder and Du [4] proposed a (rank-preserving) spectral filtering technique for multiecho MRI that operates on overlapping image subregions. In this work, we propose that a similar restriction to only local operations can potentially overcome some of the challenges faced by LR promoting methods when the row and column dimensions of the Casorati matrix differ significantly. This generalization of the LR promoting image series reconstruction paradigm, which we call Locally Low Rank (LLR) image recovery, spatially decomposes an image series estimate into a (redundant) collection of overlapping blocks and promotes that each block, when put into Casorati form, be independently LR.

**Methods:** Without loss of generality, only the recovery of fully-sampled but noisy dynamic series will be considered here. Recall the rank-constrained estimation problem:

$$u = \arg \min_u \left\{ \tau \cdot \text{rank}(C(u)) + \frac{1}{2} \|u - g\|_2^2 \right\} \quad (1)$$

where the operator  $C(u)$  places  $u$  into Casorati form,  $g$  is the observed (noisy) dynamic series, and  $\tau$  is a regularization parameter. As generalizations of (1) (e.g. for undersampled reconstruction) are often intractable, we instead consider a convex relaxation of (1) that has also been proven to promote low-rank solutions [5]:

$$u = \arg \min_u \left\{ \tau \|C(u)\|_* + \frac{1}{2} \|u - g\|_2^2 \right\} \quad (2)$$

where, for some  $m \times n$  matrix  $X$  with singular values,  $\sigma_i$ , the nuclear norm is defined as

$$\|X\|_* = \sum_{i=1}^{\min(m,n)} \sigma_i \quad (3)$$

Following earlier discussion, we propose the following generalization of (2):

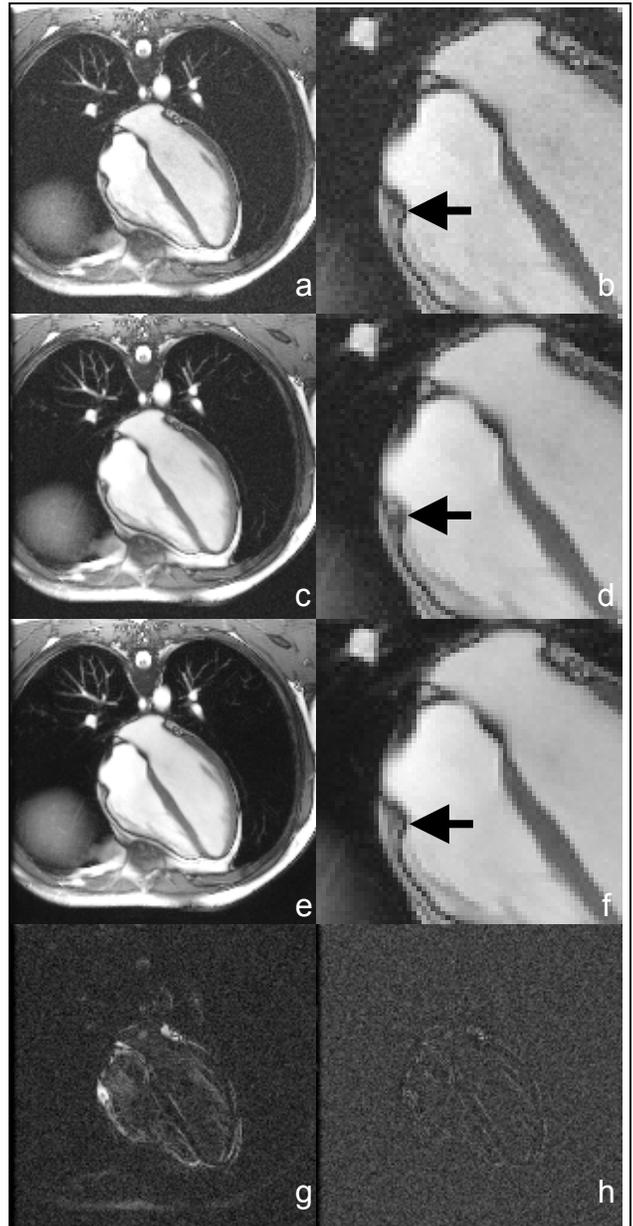
$$u = \arg \min_u \tau \sum_{b \in \Omega} \|C(R_b u)\|_* + \frac{1}{2} \|u - g\|_2^2 \quad (4)$$

where the operator  $R_b$  extracts the  $b^{\text{th}}$   $B \times B \times T$  block from the set  $\Omega$ . Define  $X = C(R_b u)$  and  $Y = C(R_b g)$ , and let  $\text{SVD}(Y) = U \Sigma V^*$  and  $\sigma_{(i,j)} = [\Sigma]_{(i,j)}$ . Each image block can be reconstructed independently by a process called singular value thresholding (SVT) [6], namely  $X = \text{SVT}(Y) = U \Sigma_{\tau} V^*$ , where the diagonal matrix  $[\Sigma_{\tau}]_{(i,j)} = \max(\sigma_{(i,j)} - \tau, 0)$ . Following recovery and reformation into their nominal dimensions, blocks are accumulated (via weighted average) to form the resulting image estimate. Assuming that the standard deviation of the (complex AWGN) series noise,  $\epsilon$ , is *a priori* known, we estimate the threshold parameter using the Marcenko-Pastur (asymptotic) upper bound on singular values of an i.i.d. random Gaussian matrix, namely  $\tau = \epsilon \sqrt{2 * (\sqrt{\max(B * B, T)} + \sqrt{\min(B * B, T)})}$  [7].

**Example:** Figure 1 shows a comparison of local versus global LR promoting reconstructions for a  $160 \times 160 \times 20$  long axis gated cardiac exam acquired on a GE 1.5T Signa Scanner (v.14.0) using a FIESTA sequence (FA=50°, TR/TE=2.8ms/1.2ms) and an 8 channel upper body coil. Noise standard deviation was estimated as  $\epsilon=10$ . For the LLR reconstruction,  $B=8$  was used and  $\Omega$  was the set of all overlapping blocks in the image space ( $|\Omega|=23409$ ). For the global LR recovery,  $B=160$  was used ( $|\Omega|=1$ ). Each coil image was reconstructed independently and subsequently combined via sum-of-squares. Reconstruction SNR was estimated using ROIs in the left ventricle and in the ex-vivo background, and were 8.71, 16.14, and 42.46 for the raw data, global LR reconstruction, and LLR reconstruction, respectively. Observe in Fig. 1a-c that, in dominantly static image regions, both the global and proposed local reconstructions managed to preserve small, low-contrast objects such as secondary pulmonary vessel branches as well as improve their conspicuity. However, note that the LLR reconstruction tended to both remove a higher degree of noise and maintain greater fidelity to the original image series that did the standard approach. These trends are obviated in the both difference (between the raw and reconstruction) images in (g-h) as well as the single frame zoom images in (b,d,f).

**Discussion:** We have proposed an extension of the low-rank promoting image reconstruction paradigm for dynamic MRI series that simultaneously offers improved noise suppression and reduced information loss. The generalization of the LLR idea to the problem of undersampled reconstruction is also straightforward (e.g., following [6]), yet still to be demonstrated. Similarly, considering the work [3], the LLR paradigm can also be extended to incorporate spatial sparsity constraints akin to those in [8,9].

**References:** [1] J. Haldar and Z. Liang, Proc. ISBI 2010; [2] S. Goud et al., Proc. ISBI 2010; [3] A. Majumdar and R. Ward, MRI, in press; [4] M. Bydder and J. Du, MRI 24(7):849-856, 2006; [5] B. Recht et al., SIAM Review 52(3):471-501, 2010; [6] J. Cai et al., SIAM J. Opt. 20(4):1956-1982, 2010; [7] M. Rudelson and R. Vershynin, Proc. ICM 2010; [8] O. Guleryuz, IEEE TIP 16(12):3020-3034, 2007; [9] M. Elad and M. Aharon, IEEE TIP 15(12):3736-3745, 2006.



**Fig. 1: Single frame from a 20-frame long-axis cardiac sequence. (a,c,e) are the original data, standard low-rank reconstruction, and LLR reconstruction, respectively. (b,d,f) are corresponding enlargements. Note the temporal blurring in the global LR reconstruction near the location of the tricuspid valve. (g) is the difference image between the global LR reconstruction and the original data, and (h) is the difference image between the LLR reconstruction and the original data. (a-f) are all windowed and leveled identically, as are (g) and (h). Other frames from the dynamic series exhibited similar characteristics.**