A novel Diffusion Kurtosis Imaging system using heteroscedastic multiple regression

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Introduction

Diffusion Kurtosis Imaging (DKI) (1) was developed to measure the non-Gaussian degree of water diffusion. In a DKI experiment, multiple b values with the maximum of around 2000 s/mm^2 are required to estimate the second-order signal decay, which is related to kurtosis. However, the signal to noise ratio (SNR) decreases as the b value increases, leading to biased estimation. In this work, we incorporate the resultant heteroscedasticity through a weighted multiple regression. Our results show that this method can significantly improve the accuracy of estimation of Mean Kurtosis (MK).

Theory

In MRI experiments, including DTI/DKI, the image is the magnitude of the complex signal (2). The measured signal $\tilde{s}$ is then a random variable

$$\tilde{s} = ((s_r + x)^2 + (s_i + y)^2)^{1/2} \qquad \text{(Eq.1)}$$

where s_r and s_i are the real and imaginary part of signal s, and x and y are independent and identically distributed as N(0, σ^2), which results in a Rician distribution for $\tilde{s}$. When the signal dominates, $\tilde{s}$ can be approximated as mean=s with a variance = σ^2. Then the measured signal in DKI is:

$$\widetilde{S_n} \approx S_0 \exp\left(-bD_n + \frac{1}{6}b^2 D_n^2 K_n\right) + \eta \qquad \text{(Eq.2)}$$

where n indexes the gradient direction, D_n is directional diffusivity, K_n is directional kurtosis, and η is background noise with constant variance σ^2. Based on this, the directional diffusion and kurtosis coefficients can be estimated by nonlinear fitting, from which the diffusion and kurtosis tensor can be calculated afterwards. Alternatively, the diffusion and kurtosis tensor can be directly estimated in tensor expression of logarithm signals:

$$\ln\left(\widetilde{S_n}\right) \approx \ln(S_0) - \mathrm{b}\sum_{i=1}^{3}\sum_{j=1}^{3} n_i n_j D_{ij} + \frac{1}{6}\mathrm{b}^2\left(\frac{1}{3}\sum_{i=1}^{3} D_{ii}\right)^2 \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} n_i n_j n_k n_l W_{ijkl} + \varepsilon \qquad \text{(Eq.3)}$$

where n_i is the component of the gradient direction vector *n*, D_{ij} is the diffusion tensor, W_{ijkl} is the kurtosis tensor, and ε is background noise with $\mathrm{var}(\varepsilon) \approx \sigma^2/S_n^2$. The model can be mathematically interpreted as a multiple regression problem with heteroscedastic error. Consequently, there are two methods to estimate DTI/DKI parameters: 1) a nonlinear fitting for each direction based on (Eq. 2); 2) weighted multiple regression based on (Eq.3). In addition, by the definition of excessive kurtosis, we can derive that the theoretical lower bound of kurtosis is K_{min} = -3.

Experiments

Imaging: The experiments were performed on a Philips 3T Achieva MRI scanner (Philips Healthcare, Best, The Netherlands) under an approved Institution Review Board (IRB) protocol. A standard 32-directions DTI sequence was used for its well-defined diffusion time. One DKI experiment consisted of four scans, where in each scan of one b value (b= 500, 1000, 1500, 2000 s/mm^2) thirty-two diffusion weighted volumes and one b0 volume was acquired. The other imaging parameters were: TE = 85 ms, TR = 3312 ms, FOV = 224 x 224 mm^2, voxel size = 1.5 x 1.5 x 1.5 mm^3, 20 axial slices, SENSE factor 2, NSA = 2. The same imaging protocol was repeated three times for calculation of the mean and standard deviation. Two ROI s of White Matter (WM) and Gray Matter (GM) highlighted in Fig. 1 were selected to present our results: WM, ROI size 5 voxels; GM, ROI size 6 voxels.

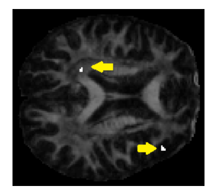

Fig. 1: selected ROIs

Simulation: The DKI data for a point phantom of WM within ROIs was simulated in MATLAB: 32 directions, five b values (b = 0, 500, 1000, 1500, 2000 s/mm^2), SNR = 20 in b0 as estimated from imaging data, and 1000 trials. Then three different methods were applied to the data: 1) **SingleDir**, the nonlinear fitting for each direction using "nlinfit" function in MATLAB; 2) **GlobalUnweighted**, the global fitting using "lscov" function in MATLAB without weighting function; 3) **GlobalWeighted**, the global fitting using "lscov" with weighting function.

Results and conclusion

In the simulation results (Fig. 2), the GlobalWeighted method has the closest mean MK to the real value and smallest standard deviation, compared with the other two methods which significantly underestimate MK. The simulated results under lower SNR show that as SNR decreases, the GlobalWeighted method continues to have the most robust performance while the other two methods deteriorate dramatically. Especially, the SingleDir method resulted in an unreasonable mean MK = -14.9249 < K_{min} at SNR= 15 (data not presented). The results from imaging data are similar to the simulated results. As shown in Fig. 3, the GlobalUnweighted method underestimate MK for both WM and GM ROIs compared with the GlobalWeighted, and its standard deviation are almost twice that of the GlobalWeighted. All the results demonstrate that the weighted global-fitting method provides a significantly more accurate and robust estimation of DKI parameters.

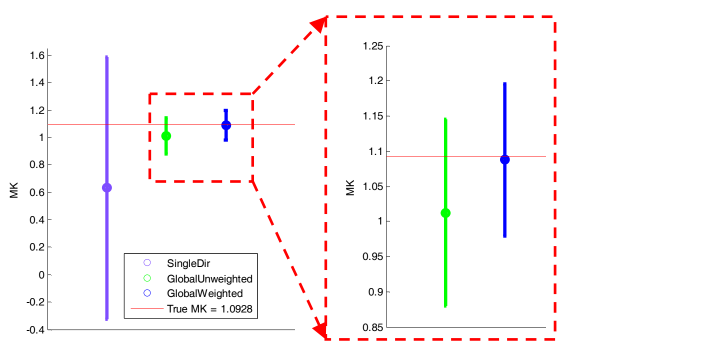

Fig. 2: Error bar (mean ± STD) of estimated MK for point WM phantom, N=1000

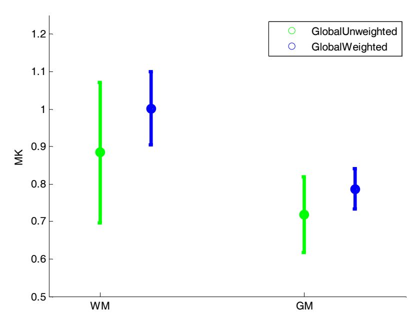

Fig. 3: Error bar (mean ± STD) of estimated MK for WM and GM ROIs.

Reference

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