A Full Bi-tensor Neural Tractography Algorithm Using the Unscented Kalman Filter

S. Lienhard[1], J. Malcolm[2], C-F. Westin[3], and Y. Rathi[2]
[1]Information Technology and Electrical Engineering, ETH Zürich, Zürich, Switzerland, [2]Harvard Medical School, Psychiatry Neuroimaging Laboratory, Boston, MA, United States, [3]Harvard Medical School, Laboratory of Mathematics in Imaging, Boston, MA, United States

Introduction: Diffusion Tensor Imaging (DTI) is an established and widely used method for analyzing neural pathways in human brains. We describe a technique that uses tractography to visualize the neural pathways by extending an existing framework which uses overlapping Gaussian tensors to model the signal. At each fiber position an unscented Kalman filter is used to find the most consistent direction as a mixture of previous estimates and of the local model. The used framework was introduced by Malcolm et al. [1, 2], and in their fiber model the diffusion tensor was described by is a spheroid, i.e. the diffusion tensor's second and third eigenvalues were identical. We extend the tensor representation so that the diffusion tensor is represented by an arbitrary ellipsoid. Tests on synthetic images show a reduction of the angular error at fiber crossings and branching and experiments with in vivo data show that our new approach finds fibers where the existing simpler model stops.

Methods: We model the diffusion-weighted signal as a mixture of two Gaussian tensors: $S_i = S_0(0.5\exp(-b\boldsymbol{u}_i^T\boldsymbol{D}_1\boldsymbol{u}_i) + 0.5\exp(-b\boldsymbol{u}_i^T\boldsymbol{D}_2\boldsymbol{u}_i))$, where $\boldsymbol{u}_i$ is the ith gradient direction, $\boldsymbol{D}_1$, $\boldsymbol{D}_2$ are the diffusion tensor matrices, b an acquisition-specific constant and S_0 is the signal without diffusion sensitizing. Each tensor matrix can be written as $\boldsymbol{D} = \lambda_1\boldsymbol{m}\boldsymbol{m}^T + \lambda_2\boldsymbol{p}\boldsymbol{p}^T + \lambda_3\boldsymbol{q}\boldsymbol{q}^T$, where λ_1, λ_2, λ_3 are the eigenvalues of $\boldsymbol{D}$ in decreasing order, i.e., $\lambda_1 \geq \lambda_2 \geq \lambda_3$, and $\boldsymbol{m}$, $\boldsymbol{p}$, $\boldsymbol{q}$ form the orthonormal basis of the diffusion ellipsoid. $\boldsymbol{D}$ has 12 free model parameters (3 eigenvalues and 3 components each of the 3 directions). Instead of using three orthonormal directions, the orientation of the tensor can also be described with 3 Euler angles, φ, θ, ψ. Using singular value decomposition the diffusion tensor matrix can be written as $\boldsymbol{D} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^T$, where $\boldsymbol{Q}$ is rotation matrix (whose columns are $\boldsymbol{m}$, $\boldsymbol{p}$, $\boldsymbol{q}$) and $\boldsymbol{\Lambda}$ is a diagonal matrix whose diagonal contains λ_1, λ_2, λ_3. Equations for obtaining the Euler angles from $\boldsymbol{Q}$ and for creating $\boldsymbol{Q}$ from the Euler angles are generally known (see [3]). The state in the unscented Kalman Filter is then represented as $\boldsymbol{x} = [\varphi_1\ \theta_1\ \psi_1\ \lambda_{11}\ \lambda_{21}\ \lambda_{31}\ \varphi_2\ \theta_2\ \psi_2\ \lambda_{12}\ \lambda_{22}\ \lambda_{32}]$. So for each of the two tensor we store the three Euler angles and corresponding three eigenvalues.

Our approach traces neural fibers by using estimations from previous points to guide the estimation at the current point. In a loop, the unscented Kalman filter recursively updates the underlying local model parameters. Each iteration predicts the new state. We assume identity dynamics for the transition function since the fiber configuration does not change drastically from one point to the next. The observation function calculates how the signal appears given a particular state. The measurement is the diffusion-weighted signal coming from scanner, S_i. Once a new estimate is calculated we move forward one small step in the principal diffusion direction $\boldsymbol{m}$ (first column of the rotation matrix $\boldsymbol{Q}$, and $\boldsymbol{Q}$ is calculated from the Euler angles in the state) of the tensor which is more consistent (the tensor whose principal diffusion direction is more aligned with the incoming direction). The loop is continued until the estimated model appears isotropic.

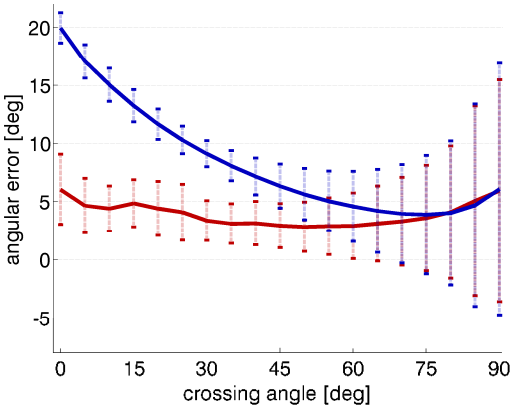

Figure 1. Angular error in crossing region.

Experiments: We created artificial scenarios of fiber crossings of different angles, we seeded in a region with only one fiber present and let the tract run into the crossing area where the resulting tensors are compared to the ground truth. Figure 1 depicts the crossing angle versus the average angular error (the difference in angle between the principal diffusion directions of the tensors) in the crossing region, blue shows the simpler model and red our new approach. The new model performs better, especially for low crossing angles, the angular resolution is higher.

In Fig. 2 below, we show the fiber tracts obtained from seeding in the right Thalamus of an in vivo scan. The blue fibers were obtained with the existing simpler model, whereas the red fibers result from our new model. The close-ups show that the new approach finds more fibers.

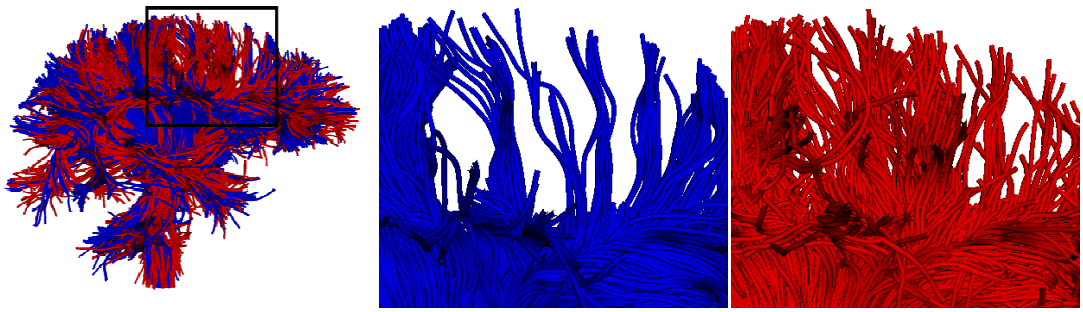
Figure 2. Result of the tractograhy.

References: [1] Malcolm, J.G., Shenton, M.E., Rathi, Y.: Neural Tractography Using an Unscented Kalman Filter. Information Processing in Medical Imaging (IPMI) 21, 2009. [2] Malcolm, J.G., Shenton, M.E., Rathi, Y.: Filtered Multi-tensor Tractography. IEEE Trans. on Medical Imaging 29, 2010. [3] Koay, C.G., Chang, L.C., Pierpaoli, C., Basser, P.J.: Error Propagation Framework for Diffusion Tensor Imaging via Diffusion Tensor Representations. IEEE Trans. on Medical Imaging 26, 2007.