

A numerical advection-diffusion model to fit dynamic contrast-enhanced MRI (DCE-MRI) data

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Purpose

Both the geometry of a system as well as the fluid flowing mechanisms plays a fundamental role in the modeling of mass transport. Various anatomical models of mass transport in tumors have been developed. Their complexity ranges from 3D models to axially distributed blood-tissue exchange models with an advection-diffusion equation that is simplified to various extents [1]. The choice of the model has an impact on the mass transport characteristics which are derived therefrom. Analysis of DCE-MRI data mostly relies on the simplest versions of these models, i.e. on compartmental models based on one or two well-mixed compartments (plasma and interstice) with diffusion as the prevailing mechanism of mass transport [2]. Therefore, a prospective study was undertaken to test a model which does include advection mechanism and assumes that fluid flow in tissues can be described by Darcy's law in a porous media, to fit DCE-MRI data.

Methods

■ **Computational Model:** The advection and diffusion of the MR contrast agent in the tumoral tissue is modeled by:

$$\frac{\partial C}{\partial t} + u \cdot \nabla C - D \nabla^2 C = \phi \quad \text{eq.(1)}$$

where C is the concentration of the contrast agent, u is the advective velocity vector field, D is the molecular diffusion coefficient in the tissue and ϕ is a source/sink term. The numerical solution for $C(x,t)$ results from a finite element approach [3] that consists to rewrite the mathematical model eq.(1) in a set of algebraic relations eq.(2), applied on a mesh made of elementary segments. The originality of the proposed approach is to assimilate each segment to a dedicated finite element in order to benefit from a well-known numerical implementation. Thus, the set of segments is a mesh composed of n nodes (cells) and e elements (connections). The global process may then be decomposed on three phases. Phase 1, the pressure $p(x_i)$ on each node i of the mesh based on mass flow (q_i) conservation is calculated.

$$\sum_i q_i = 0 \Rightarrow \sum_i \frac{L_p S_{ij}}{L_{ij}} (p_i - p_j) = 0$$

Phase 2, the velocity distribution $u(x_i)$ is deduced from the nodal pressure $p(x_i)$ by solving Darcy's law, $u = -(L_p S / \mu) \nabla p$, where the viscosity μ is assumed to be homogeneous. Phase 3, the nodal concentration $C(x_i, t)$ is calculated from an implicit temporal resolution (for numerical stability) of the set of algebraic equations issued from eq.(1).

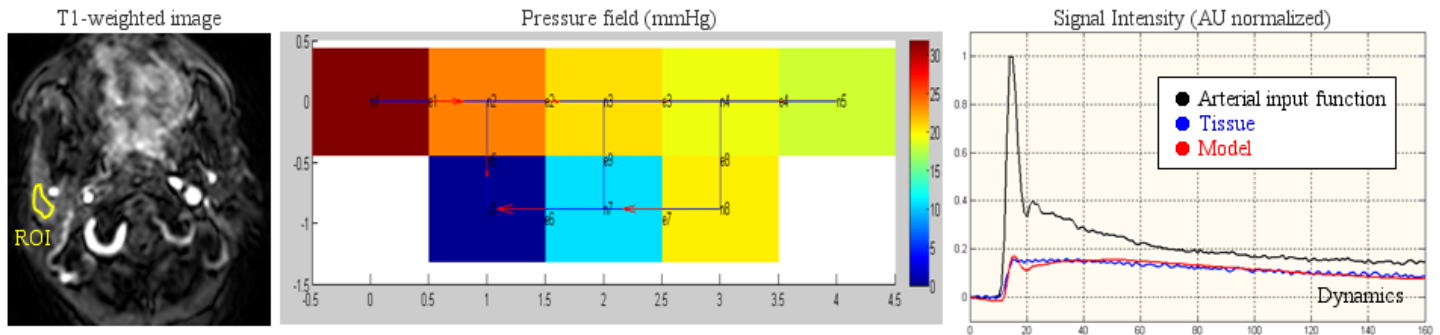
$$[M] \{ \dot{C}(t) \} + ([K_{advection}] + [K_{diffusion}]) \{ C(t) \} = \{ \phi \} \text{ with } \{ C(0) \} = \{ C_0 \} \quad \text{eq.(2)}$$

■ **MRI:** The data set was obtained from a patient with a squamous cell carcinoma of the head and neck region. A transversal T1-weighted spoiled 3D gradient echo sequence with SENSE factor 2 in the transversal plane was performed at 3T. Thirteen slices covering the full head & neck area were scanned using the following parameters: slice thickness $L = 4\text{mm}$, 224×224 matrix, in-plane voxel size = 1.96×1.75 after reconstruction, flip angle = 30° , TE = 1.1ms, TR = 11.25ms, 160 dynamics, temporal resolution = 3.25s. Patient received 2mL per 10kg of body weight of gadolinium Gd-DTPA (Magnevist[®]) followed by 30mL of saline flush injected at a rate of 2mLs^{-1} with an automated power injector. A linear relationship between signal intensity (SI) and Gd-DTPA concentration was assumed for the range of expected contrast agent concentrations within the studied tissues.

■ **Curve Fitting:** A radiologist manually drew one ROI covering the ipsilateral carotid (for the arterial input function) and another ROI in the tumoral tissue (for the tissue output). The optimization of the fit was achieved with a weighted non-linear least squares fit based on the Levenberg-Marquardt method combined with multiple starting parameter values [4]. The following parameters were optimized: L_{ij} the length of the element, $L_p S_{ij}$ the hydraulic conductivity surface area product of the element, D the molecular diffusion and ϕ the source/sink term.

Figures

Left: axial view of the perfusion sequence at $t=68\text{s}$. A ROI is depicted within the SCC. Middle: mesh composed of 5 upper segments (vascular space) and 3 lower segments (interstitial space). p values at arterial entry (node n1) and venous end (node n5) are fixed to 32mmHg and 18mmHg respectively. Red arrows represent the velocity vector field according to Darcy. Right: the AIF is imposed at n1 while the model is calculated at n5.



Results and Conclusion

This first model fits the SI versus time curve reasonably well, suggesting that the transport of the contrast media within neoplastic tissues can be described by advection and diffusion mechanisms. Further work is needed to dimension the model accurately (i.e. to optimize the number of free parameters, the values of the fixed parameters, the number of elementary segments) and assess its clinical value for DCE-MRI data analysis.

References

[1]. Beard DA, Annals of Biomedical Engineering, 2000;28:253–268. [2]. Donaldson SB, Magn Reson Med 2010;63:691–700. [3] Dhath G, Méthode des éléments finis, Hermès Science–Lavoisier Press 2005;ISBN:2-7462-0979-9. [4] Ahearn TS, Phys Med Biol 2005;50:85-92.