Theoretical determination of the dielectric constant for passive RF shimming at high field

M. L. Jayatilake[1,2], J. Storrs[1,3], W-J. Chu[1,3], and J-H. Lee[1,4]
[1]Center for Imaging Research, University of Cincinnati, Cincinnati, OH, United States, [2]Department of Physics, University of Cincinnati, Cincinnati, OH, United States, [3]Department of Psychiatry and Behavioural Neuroscience, University of Cincinnati, Cincinnati, OH, United States, [4]School of Energy, Environmental, Biological, and Medical Engineering, University of Cincinnati, Cincinnati, OH, United States

Introduction: Optimal image quality for MRI at high fields requires a homogeneous RF (B_1) field among others; however, dielectric properties of the human body result in severe B_1 field inhomogeneity. These are resulted from constructive and destructive RF interactions of complex wave behaviour, which become worse with increasing Larmor frequency. Placement of a shim object with high-dielectric constant adjacent to the body has been proposed as a method for reducing B_1 inhomogeneity by altering wave propagation within the volume of interest [1, 2, 3, and 4]. Selecting the appropriate permittivity and the quantity of material for the correction of B_1 field is essential. While previous work determined primarily empirically the dielectric properties of the shim object, this work introduces a theoretical framework for calculating the requisite dielectric constant of the passive shim material and verifies the accuracy using simulated field maps.

Theory and Methods: The human head is approximated by a cylinder with radius r_1 of 20 cm. The head lies inside a TEM coil with inner and outer radii r_2 and r_3 of 28 and 34 cm, respectively. The head and coil are coaxial (Fig. 1). The relative permittivity and conductivity of brain is $\varepsilon_r = 58$ and $\sigma = 0.3$S/m, respectively; and both the brain-to-coil and inner-to-outer coil spaces are filled with air with permittivity ε_0 [2]. It is assumed that B_1 field propagates along the B_0 direction. The general solution of the propagating electromagnetic wave has a unique solution at each region with unknown amplitudes. After applying the boundary conditions to wave solutions in each region, the characteristic equation can be determined [5]. The characteristic equation can further be simplified by substituting the first terms in series expansion of the Bessel function. The simplified characteristic equation is

$$f(k_{\rho0},k_\rho,k_0,k,k_z)=r_1^2\cdot k_z^2\cdot k_\rho^2\cdot\left(k^2-k_0^2\right)\cdot\left(r_2^2-r_1^2\right)\cdot\alpha+k_\rho^2\cdot k_{\rho0}^2\cdot\left(2k^2\cdot k_{\rho0}^2\cdot\left(r_2^2-r_1^2\right)+k_\rho^2\cdot k_0^2\cdot\left(r_2^2+r_1^2\right)\right)\cdot\left(r_1^2\cdot k_z^2\cdot k_\rho^2\cdot\alpha'-k_{\rho0}^2\cdot\alpha\right)=0 \quad (1)$$

Here k, k_0, k_z and k_ρ are the brain, air, axial and radial propagation constants respectively. The k_z and k_ρ characterize the RF field amplitude distribution within the brain, where α and α' are given below.

$$\alpha=-2\cdot\left(r_1^2+\pi\cdot r_2\right)/\pi^2\cdot r_1\cdot r_3^3\cdot k_{\rho0} \quad (2);\qquad \alpha'=-2\cdot\left(r_1+\pi\cdot k_{\rho0}\cdot r_3^2\right)/\pi^2\cdot r_1\cdot r_2\cdot r_3^2\cdot k_{\rho0} \quad (3)$$

The values of k_ρ and $k_{\rho0}$ depend on the dielectric properties within the brain and brain-to-coil region.

$$k_\rho^2=k^2-k_z^2 \quad (4);\qquad k_{\rho0}^2=k_0^2-k_z^2 \quad (5)$$

Loading with high dielectric material into the brain-to-coil air region significantly influence on the propagation characteristics of the head coil which is demonstrated by changes in k_z. Combining Eqs. (2), (3), (4) and (5), the $f(k_{\rho0}, k_\rho, k_0, k, k_z)$ can be expressed as a function of k_z and k_ρ. The modified characteristic equation can be factored to express the k_z as a function of the k_ρ. Based on this relationship between radial and axial propagation constants, it is possible to maximize k_z and minimize k_ρ. Additionally, the $f(k_{\rho0}, k_\rho, k_0, k, k_z)$ can also be expressed as a function of k_z and effective dielectric constant (ε) in the body-to-coil region, i.e. $g(k_z, \varepsilon)$. The range of required effective ε and corresponding k_z can then be determined when the modified characteristic function $g(k_z, \varepsilon)$ approaches zero. Once the maximal k_z has been determined, the requisite effective ε for B_1 passive shimming can be determined.

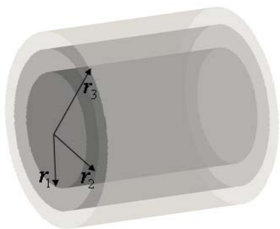

Figure 1 Cylindrical model of brain, coil and RF shield.

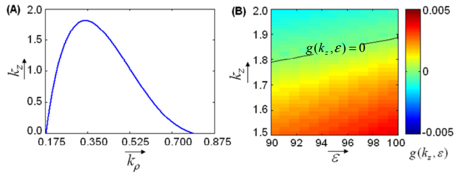

Figure 2(A) Plot of the correlation of axial vs. radial propagation constant. (B) The distribution of the modified characteristic equation of $g(k_z,\varepsilon)$ according to variation of both axial propagation constant and dielectric constant.

Results: Based on the relationship between k_ρ and k_z (Fig. 2A) the minimum k_ρ of $0.34cm^{-1}$ is obtained at the maximum k_z of $1.82cm^{-1}$ for a 4T magnet. Additionally, at the first maximum of k_z, the radial wave length of $\lambda_\rho = 2\pi / k_\rho$ is about 18.5*cm* which is comparable to the size of the subject's brain. According to Fig. 2B the appropriate magnitude range of the effective ε of 92 - 94 is determined corresponding to k_z of $1.82cm^{-1}$ as $g(k_z, \varepsilon)$ reaches zero. Fig. 3 shows the simulated B_1 field map after filling the body-to-coil region with (A) air and (B) a dielectric material with ε of 94 which is obtained according to Eq. (1). The B_1 field map is normalized to the field amplitude at the center of the FOV with the assumption that the entire brain has a uniform dielectric constant. The results clearly show that the ripples of the B_1 field distribution within the brain are significantly reduced as the ε within the body-to-coil region increased.

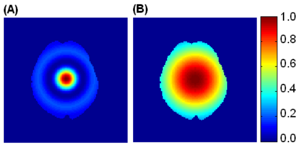

Figure 3 The RF field distributions for axial slices of the subject's brain.

Discussion: We have demonstrated with simulation data, as shown in Fig. 3, that this method provides an effective solution for selecting an appropriate material to improve B_1 field homogeneity within the human brain at 4T. This approach should be applicable to all fields. Experimental verification of the method is currently being conducted *in vivo*.

References: [1] Haines et al., J Magn Reson Imaging 203;323-327 (2010). [2] Foo et al., Magn Reson Med 23:287– 301(1992). [3] Yang et al.,J Magn Reson Imaging 24;197–202(2006). [4] Yang et al., J Magn Reson Imaging 47;982–989(2002). [5] Foo et al., Magn Reson Med 21:165–177(1991).