A novel coil design method for manufacturable configurations at optimal performance

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Introduction

Over the past 20 years many papers have discussed theoretical design methodologies for MRI gradient or shimming coils. Poole et al. [1] presented a method based on minimax current density to design a coil. The current-carrying surface is discretized using a triangular mesh, and the current density value on the surface can be obtained by optimizing a cost function related to the desired magnetic field and the maximum of current density. This is an attractive method because minimizing the maximum of the current density can help increase the minimal distance between adjacent conductors of the designed coil. This increase can improve the producibility of the coil due to limited size of the conductor and reduce local hot spot of the coil. However, the usage of maximum of current density as an objective function can also increase the total number of iterative optimization steps, typically 10,000-100,000 [1], because the maximum of current density is not differentiable.

Based on an alternative differentiable objective function which is related with the maximum of current density, an iterative optimization method is proposed to design a biplanar micro-coil in this abstract. The main goal of this method is to increase the minimal distance between adjacent conductors of the coil. The planar surface can be discretized using a high-order triangular mesh. The magnetic field is calculated using the Biot-Savart law where the surface current density is expressed using a stream function and the surface integration is implemented using surface numerical integration based on the shape functions of the finite element.

Theory

The design purpose of the biplanar micro-coil is to obtain required magnetic field distribution in a region of interest (ROI) while keeping the minimal distance between adjacent conductors of the micro-coil as big as possible. In order to obtain required magnetic field distribution in the ROI, the first objective is chosen as

$$F_1 = \sum_{i=1}^{k} (B_z(x_i, y_i, z_i) - B_{des,z}(x_i, y_i, z_i))^2, \quad (1)$$

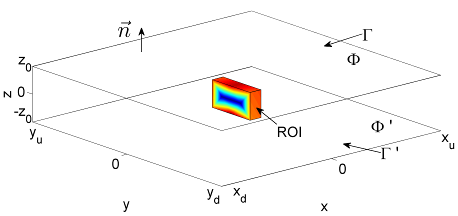

Figure 1. current-carrying surface and ROI.

where $B_{des,z}(x_i, y_i, z_i)$ is the z-component of the desired magnetic field, (x_i, y_i, z_i) are the coordinates of the sampling points in the ROI. The magnetic field B_z is calculated using the Biot-Savart Law:

$$B_z(x_i, y_i, z_i) = \frac{\mu_0}{4\pi}\int_{y_d}^{y_u}\int_{x_d}^{x_u} \frac{J_y(x,y)(x_i - x) + J_x(x,y)(y_i - y)}{((x_i - x)^2 + (y_i - y)^2 + (z_i - z_0)^2)^{3/2}} + \frac{J'_y(x,y)(x_i - x) + J'_x(x,y)(y_i - y)}{((x_i - x)^2 + (y_i - y)^2 + (z_i + z_0)^2)^{3/2}}\, dxdy, \quad (2)$$

where J_x (J'_x)and J_y (J'_y)are the x and y components of the surface current density on the upper(lower) current-carrying planar surface Γ (Γ') respectively(Figure 1), μ_0 is the magnetic and z_0 is the distance between the upper surface from and the coordinate origin. The surface current density can be expressed using the stream function Φ (Φ') as:

$$\vec{J} = \nabla \times (\Phi \vec{n}) = (0, \partial\Phi(x,y)/\partial y, -\partial\Phi(x,y)/\partial x)^{\mathrm{T}}, \vec{J}' = (0, \partial\Phi'(x,y)/\partial y, -\partial\Phi'(x,y)/\partial x)^{\mathrm{T}} \quad (3)$$

where $\vec{n}$ is the unit vector normal to the planar surface. The current-carrying surface is discretized using a triangular mesh and the value of the stream function is interpolated using the shape function Ψ_j of the Argyris element [2], i.e. $\Phi=\Sigma_j \alpha_j\Psi_j$ where α_j are coefficients. The micro-coil optimization is an inverse problem. Generally, one needs to use a regularization technique to obtain a unique solution. In order to implement the regularization effect, the dissipated power of the coil is chosen as the second objective, i.e. $F_2 = (\int_\Gamma |\vec{J}|^2 d\Gamma + \int_{\Gamma'} |\vec{J}'|^2 d\Gamma')/(2t\sigma)$, where t denotes the thickness of the surface and σ is the electrical conductivity of the surface. In order to obtain a big minimal distance between adjacent conductors of the coil, the third objective is chosen as $F_3 = (\int_\Gamma |\vec{J}|^p d\Gamma)^{1/p} + (\int_{\Gamma'} |\vec{J}'|^p d\Gamma')^{1/p}, p > 2.$ That means that the third objective is the formula of the p-norm of surface current density which is differentiable and can converge to the infinite norm of surface current density when p tends to infinity. In the paper, the value of p is set to 5.

The weighted sum approach is used to solve the multi-objective optimization problem. In this approach, the problem is converted to minimize the single scalar objective function as $\alpha F'_1 + \beta F'_2 + \delta F'_3$ where F'_1, F'_2, F'_3 denote the normalized objective functions corresponding to F_1, F_2, F_3, α, β and δ are the weights of the respective normalized objective function, and the sum of all weights is one, i.e. $\alpha+\beta+\delta=1$. Then, this converted single objective optimization problem can be solved by L-BFGS method [3].

In order to avoid a bad local minimum from an arbitrary initial value, the whole optimization procedure has been split to two stages. In the first optimization stage, we mainly focus on the optimization of objective functions F_1 and F_2, i.e. $\delta=0$, in order to generate a user-specified magnetic field distribution in ROI. The optimal solution in this stage is used to an initial value in the second optimization stage. In the second optimization stage, all three objective functions are optimized to obtain a final optimal solution which can be used to construct a micro-coil with big minimal distance between adjacent conductors of the coil.

Numerical results

Figure 2 shows an example of a micro-coil design on the biplanar surfaces with $x_d = y_d = -2.5$ mm, $x_u = y_u = 2.5$mm and $z_0 = 0.5$mm (Figure 1). The ROI is a solid block with length = 0.25 mm, width = 1 mm and height=0.5 mm (Figure 1). The desired magnetic field is a homogenous field with $B_{des,z} = 0.2$ mT. The total number of iteration steps in the first and second optimization stage is less than 1000.

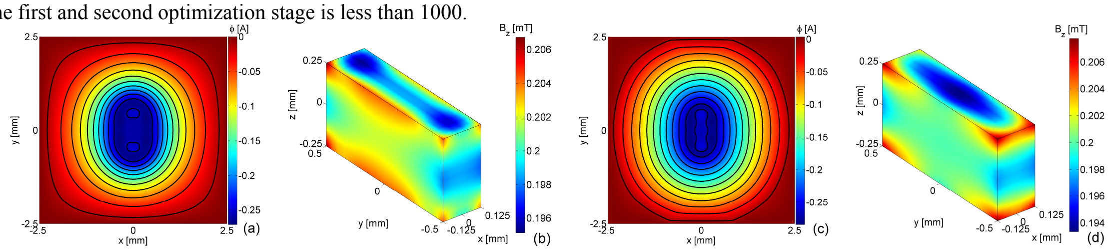

Figure 2, the optimal stream function and its contour lines (coil layout) on the upper and lower surface in the first (a) and second (b) optimization stage. (b) and (d) Numerical magnetic field B_z generated by the designed coils based on the coil layouts in (a) and (b) respectively.

Discussion and outlook

This abstract presents an iterative optimization method to design a biplanar coil for MRI. Based on an auxiliary new objective, i.e. p-norm of current density, one example demonstrates that this method can be used to design a micro coil in which the minimal distance between two adjacent conductors can be increased.

Reference

1 Poole et al., J. Phys. D: Appl. Phys., 2010, 43, 095001.
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