The "Central Signal Singularity" Phenomenon in Balanced SSFP

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Introduction: Small, alternating perturbations of parameters in a balanced SSFP (bSSFP) sequence can cause large, highly localized deviations in the magnetization profile at specific frequencies in the center of the passband. We show that these deviations correspond to singularities in the steady-state signal equation, and we present mathematical and physical explanations of this phenomenon. We demonstrate the phenomenon via Bloch simulations and phantom imaging and discuss potential applications for positive-contrast imaging and fMRI. This phenomenon was first investigated in the context of wideband SSFP [1], where it was denoted "Central Signal Dip" and was caused by the perturbation of repetition times (TRs). For simplicity, we present a theory and analysis of the central signal singularity for the case of flip-angle perturbations. These techniques can be easily generalized to other parameters such as RF phases or TRs.

Theory: If alternate flip angles of a bSSFP sequence are perturbed by Δα degrees, the resulting magnetization profile has sharp notches and spikes in the odd and even TRs, respectively (Fig. 1). Since the sequence repeats after two TRs, the steady-state magnetization is $M_{ss}=(I-R)^{-1}b$, where I is the 3x3 identity matrix, R is a 3x3 matrix accounting for excitation, precession, and relaxation during the two TRs, and b is a 3x1 vector accounting for relaxation. The analysis can be simplified using the eigenvector decomposition of R: $R=V\Lambda V^{-1}$, which yields $M_{ss}=V(I-\Lambda)^{-1}V^{-1}b$ [2]. The steady-state magnetization can then be viewed as a weighted sum of components directed along each eigenvector (v_i), where the weights are elements of the diagonal matrix $(I-\Lambda)^{-1}$. If one eigenvalue (say, λ_1) is closer to 1 than the others, then $(1-\lambda_1)^{-1}$ will dominate, causing M_{ss} to be directed parallel to the corresponding eigenvector. Since TR<<T1,T2, R is close to a pure rotation matrix, having at least one eigenvalue that is near 1. At most frequencies, only one eigenvalue of R is close to 1, and M_{ss} smoothly "tracks" the corresponding eigenvector (Fig. 1c). However, at integer multiples of 1/TR Hz, all three eigenvalues are close to 1 (i.e., $R \approx I$), yielding a singularity in the expression for M_{ss}. In the absence of perturbations, R has one dominant eigenvalue at these frequencies, and M_{ss} continues to smoothly track the corresponding eigenvector (Fig. 1c). If sequence parameters are imbalanced (e.g., Δα=1°), then R has two dominant eigenvalues, and M_{ss} no longer tracks a single eigenvector, resulting in signal perturbations at these critical frequencies (Fig. 1d).

To gain a physical understanding of the central signal singularity, we track the trajectory of the magnetization vector for several different isochromats (Fig. 2). For Δα=1°, the steady-state trajectory is tilted to account for this one-degree imbalance in nutation. At 60 Hz and 10 Hz, small shifts of the trajectory in the x-y plane are sufficient to offset the perturbation, resulting in little change in the transverse signal. However, at 0 Hz, the steady-state trajectory is confined to the y-z plane, and a large rotation is necessary so that relaxation differences in even and odd TRs offset the one-degree imbalance in nutation.

Results: A uniform spherical phantom was scanned with a linear field gradient to verify the spectral profile of the perturbed bSSFP sequence. A 3D sequence was scanned with TR=4.6 ms, α=60°, Δα=1°, FOV=24 cm, resolution=0.9x0.9x5 mm^3. Interleaved acquisitions were obtained during even and odd TRs, resulting in a scan time of 1:53s. Cross-sectional profiles of the even- and odd-TR images closely match theory. These acquisitions can be subtracted to yield a profile with sharp peaks centered at integer multiples of 1/TR Hz and broad stopbands in between (Fig. 3).

Discussion: We have explained the central signal singularity phenomenon using linear systems theory and have shown that deviations in the profile correspond to singularities in the signal equation. Additionally, we provided a physical explanation of the phenomenon by analyzing the steady-state magnetization trajectories of different isochromats. Finally, we showed that the complex difference profile exhibits sharp peaks and broad stopbands. This complex difference technique has numerous potential applications. (1) The spectrally-selective peaks can be centered on off-resonant frequencies for positive-contrast imaging of susceptibility-induced frequency shifts [3]. The broad stopbands yield a high level of background suppression. (2) The peaks of the complex difference profile are also accompanied by sharp phase transitions, which can be used to image the BOLD-induced frequency shift in fMRI [4].

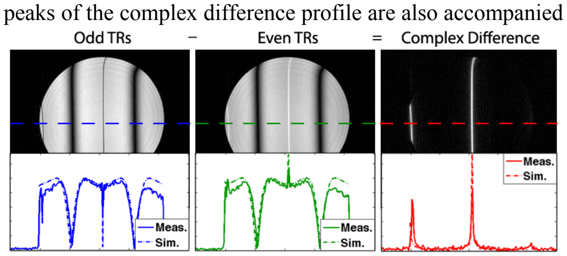

Figure 3. A ball phantom was scanned with a horizontal gradient shim. Acquisitions (Δα=1°) in odd (left) and even (center) TRs can be subtracted to yield a profile with peaks at integer multiples of 1/TR (right).

References:
[1] Nayak K, *et al.* MRM 58: 931-938, 2007. [2] Hargreaves B, *et al.* MRM 46: 149-158, 2001. [3] Çukur T, *et al.* MRM 63: 427-437, 2010. [4] Miller K, *et al.* MRM 50: 675-683, 2003.

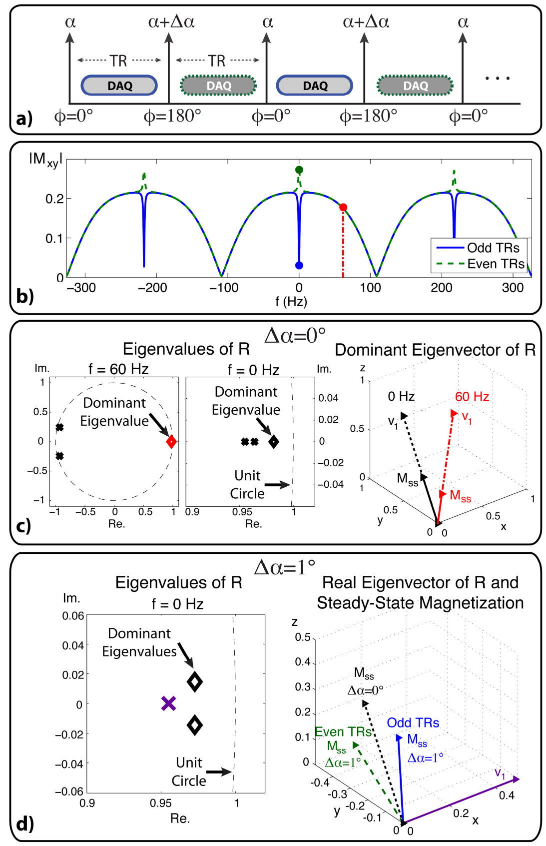

Figure 1. (a) Perturbed bSSFP pulse sequence, with alternate flip angles perturbed by Δα°. **(b)** Magnetization profile in odd TRs (solid blue) and even TRs (dashed green) for TR=4.6 ms, α=60°, Δα=1°, T1/T2=5. **(c)** Eigenvalues and eigenvectors of the system matrix, R, for a standard bSSFP sequence (Δα=0°). At 60 Hz, there is one dominant eigenvalue (red diamond), and the steady-state magnetization vector (M_{ss}) is parallel to the corresponding eigenvector. At 0 Hz, all eigenvalues are close to 1 (i.e., $R \approx I$), but M_{ss} remains parallel to the dominant eigenvector (dotted black). **(d)** Eigenvalues and eigenvectors for perturbed sequence (Δα=1°) at 0 Hz. Real eigenvalue (purple 'x') is no longer dominant, and M_{ss} in odd (solid blue) and even (dashed green) TRs deviates significantly from M_{ss} for standard bSSFP at 0 Hz (dotted black).

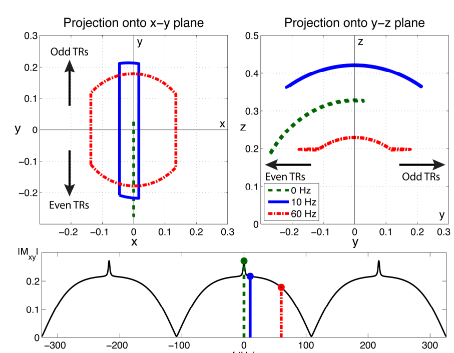

Figure 2. Projection of magnetization trajectory of perturbed sequence (Δα=1°) onto x-y and y-z planes. Trajectory at 60 Hz (dot-dashed red) is essentially identical to bSSFP. X-Y trajectory at 10 Hz (solid blue) is shifted to compensate for the unequal flip angles. Trajectory at 0 Hz (dashed green) is constrained to the y-z plane and requires large rotation to account for unequal flip angles.