Calculation of Shear Stiffness in Noise Dominated Magnetic Resonance Elastography (MRE) Data Based on Principal Frequency Estimation.

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Introduction: Magnetic resonance elastography (MRE) based calculation of shear stiffness involves mathematical processing of shear wave induced displacements encoded into the phase of the MRE image and relies upon high signal-to-noise ratio (SNR) MRE data sets. Under conditions of low SNR, such as is the case in ^{1}H lung MRE, commonly used MRE 'inversion' algorithms underestimate shear stiffness. A new inversion algorithm, based on the principal spatial frequency of the shear wave displacement map, is presented and shown to be less sensitive to decreased SNR when compared to the commonly used local frequency estimation (LFE) method [1]. Finite element simulations and ex vivo experiments were conducted so as to assess the robustness of this approach in both theoretical and experimental low SNR MRE data.

Principal Frequency Analysis (PFA) of Shear Modulus Theory: In MRE a propagating shear wave is imaged so as to acquire equally spaced temporal snap shots of the associated displacement field. Estimation of shear stiffness involves application of a temporal Fourier transform at each pixel to extract the motion at the shear wave excitation frequency. This data is then processed to provide an estimate of shear stiffness by application of a so-called inversion algorithm [2]. In the proposed method, the entire image (or some large region), filtered as above, is spatially Fourier transformed in two-dimensions. A thresholding procedure is then applied in which the peak amplitude of the spectrum is identified followed by zeroing of amplitude values that are less than some fraction of the peak value. A weighted average of the radial spatial frequencies corresponding to spectrum amplitude values above this threshold is then calculated. This average is given by $f_{centroid} = \sum f_{rad} * M(f)^2 / \sum M(f)^2$ and involves the magnitude, $M(f)$, of the spectrum at each spatial frequency above the threshold multiplied by its corresponding radial frequency f_{rad}. The principal spatial frequency identified as $f_{centroid}$ is then used to calculate the wavelength and hence shear stiffness. This approach does not attempt to calculate local stiffness values, but rather a single average value over a large region.

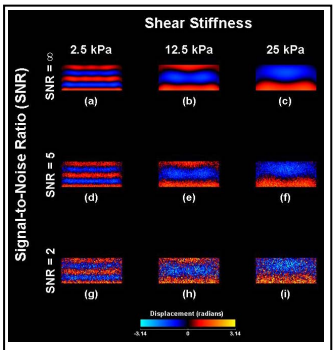

Figure 1: Finite element simulation data for a shear wave field as a function of stiffness and SNR.

Methods:

Finite Element Simulations (FEM): A wave displacement field within a uniform phantom excited with shear waves was simulated using finite element modeling (COMSOL 3.5, Burlington, MA). The finite element model consisted of a rectangular cuboid with a cross section of side 3.75 cm X 1.6 cm with infinite length, effectively providing a two dimensional model. Time-harmonic waves induced at one of the boundaries propagated uniformly within the model domain according to the Helmholtz wave equation. Wave displacement fields were simulated at three different values of shear stiffness (2.5, 12.5 and 25 kPa) of the phantom while increasing amounts of Gaussian noise (mimicking MRI noise) were added so as to have signal to noise ratios (SNR = median wave amplitude/standard deviation of noise) ranging from 9 to 2.

Ex Vivo Validation: Ex vivo ^{1}H MRE was performed on five adult female Sprague-Dawley rat lungs according to previously described experimental method [3] and in accordance with our Institutional Animal Use and Care (IACUC) guidelines. After excision en bloc each lung set was degassed and then inflated to a pressure of 20 cm H_20. The lungs were then deflated to 3 cm H_20 after which ^{1}H MRE imaging was performed. The lungs were then inflated to a pressure of 20cm H_20 for one minute followed by deflation to 6cm H_20 after which ^{1}H MRE imaging was repeated. This process was reproduced for additional inflation pressures of 9, 10, 12 and 15 cm H_20, covering a range of ex vivo SNR conditions.

Calculation of Shear Stiffness: Simulated and ex vivo wave data were processed with both LFE and PFA algorithms. Shear stiffness values obtained from the FEM data were compared to the actual stiffness values used to simulate the displacement wave fields.

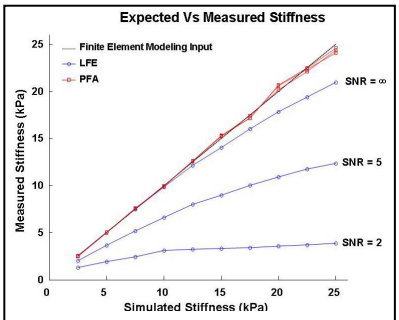

Figure 2: LFE vs PFA derived shear stiffness versus SNR.

Results:

Finite Element Simulations: Figure 1 shows the wave data obtained from the finite element simulation for three stiffness values of 2.5 kPa, 12.5 kPa and 25 kPa respectively at three separate SNR values of infinity (a-c), 5 (d-f) and 2 (g-i). The simulation demonstrates that as shear stiffness increases, the number of full wavelengths present within the model decreases. In addition, the effects of decreasing SNR are also seen by the decreased conspicuity of the shear wave front. This is quantified in Figure 2 which plots the calculated stiffness values using LFE (blue) and PFA (red) as a function of the expected stiffness used for the FEM simulation for the three SNR values. The plot also demonstrates the sensitivity of LFE to low SNR MRE data. Additionally, even at an SNR of infinity (i.e. no noise), the LFE stiffness estimates deviate from the expected stiffness due to the longer shear wavelength in comparison to the model dimensions. The expected stiffness values are also plotted as the black line, which has a slope of one.

Ex Vivo Validation: Figure 3 shows the phase-difference (wave field) and magnitude data for the last ex vivo lung set demonstrating the effects of low SNR ^{1}H lung MRE data and the decreasing SNR of both data types with lung inflation pressure. For all five data sets, the PFA method demonstrated a larger linear correlation coefficient ($0.64 \leq R \leq 0.86$) compared to the values for the LFE-based estimates ($-0.65 \leq R \leq 0.39$). The slope of the least squares linear regression fit was larger for all PFA-based estimates (range 0.72 – 1.9) than for the LFE-based estimates (range -0.22 – 0.39). LFE-based slopes were positive in only one of the five data sets. Because it is appreciated that lung shear stiffness is linearly related to inflation pressure and that for rat lung the slope is between 0.96 and 1.52 [4], PFA-based estimates of shear stiffness were found to more closely agree with previously reported experimental data.

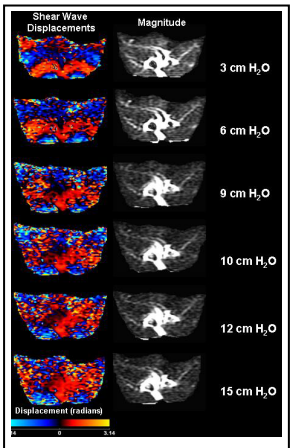

Figure 3: Ex vivo rat lung phase-difference & magnitude ^{1}H MRE data versus inflation pressure.

Conclusion: These data indicate that PFA-based methods provide more accurate estimates of shear stiffness in comparison to conventional inversion methods when the SNR of the MRE data is low.

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References:

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