Global Minimum Peak RF Design for Large Time-Bandwidth Saturation Pulse

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Introduction Saturation pulses $rf(t)$ are essential to many imaging applications [1-3]. Criteria for desirable saturation profile $|M_z|$ are flat passband and sharp profile with minimum peak $rf(t)$ amplitude. Design parameters for RF pulses include passband δ_1 and stopband δ_2 ripple tolerances and time-bandwidth product tb [1]. The well-known Shinnar-Le Roux (SLR) RF pulse design technique is a transform that relates magnetization profile to two polynomials A_N and B_N [4-6]. B_N has been obtained, in the past, by traditional digital filter design techniques using the parameters above as input. A conventional approach (for minimum-peak $rf(t)$) is to design a maximum-phase polynomial B_N, factor B_N to obtain its roots, then combinatorially search by root inversion [7] over all possible phase patterns. But this conventional method is limited to $tb \approx 18$ before number of combinations becomes prohibitive. For tb well in excess of that, we propose a novel Optimization technique that determines a B_N yielding the global minimum peak $rf(t)$ amplitude.

Methods For saturation pulses, the RF pulse $rf(t)$ and polynomial B_N are essentially the same. We first use Optimization to generate a minimum peak B_N, then the saturation profile $|M_z|$ and RF pulse $rf(t)$ are found via SLR transform as in the conventional method. We want to find a minimum peak amplitude B_N whose frequency response $H(\omega)$ satisfies design specifications, as formulated in Eq.1. But this problem statement is nonconvex (*i.e.* solution not necessarily globally optimal). So instead, define an autocorrelation matrix of B_N as $G \triangleq B_N B_N^{\mathrm{H}} \in \mathbb{C}^{N\times N}$, where G is positive semidefinite with rank 1. Summing along each of $2N-1$ subdiagonals produces entries of the autocorrelation function r of B_N, where $r \triangleq r_{\mathrm{re}} + i\, r_{\mathrm{im}} \in \mathbb{C}^N$. In particular, the main diagonal of G holds squared absolute entries of B_N (Fig.1). Minimizing $\|B_N\|_\infty$ is therefore equivalent to minimizing $\|\mathrm{diag}(G)\|_\infty$. Define $I_0 \triangleq I$ and define I_n as a zero matrix having vector $\mathbf{1}$ along the n^{th} superdiagonal when n is positive or $\mathbf{1}$ along the n^{th} subdiagonal when n is negative. By spectral factorization [8], $R(\omega) = |H(\omega)|^2$, an equivalent problem is expressed in Eq.2.

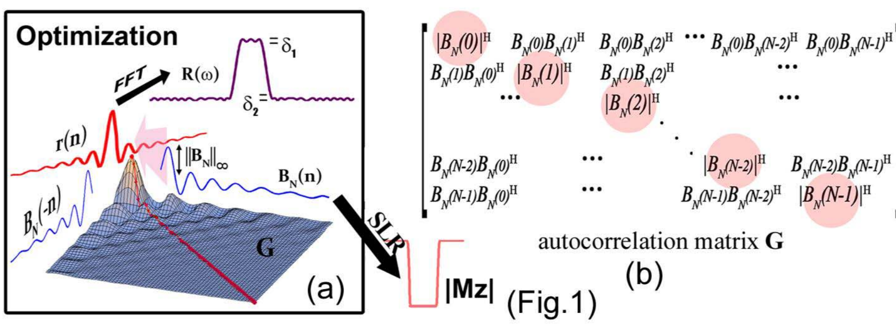

Fig.1 (a) Relationship of parameters used in optimization: Autocorrelation matrix G composed of columns of B_N whose peak amplitude is being minimized. Summing each $2N-1$ subdiagonals of G (main diagonal shown in red) gives autocorrelation function r whose magnitude spectrum satisfies design specifications. Optimized B_N gives saturation profile $|Mz|$ via SLR transform. (b) Main diagonal of autocorrelation matrix G (highlighted in red) is the Hadamard product of polynomial B_N. First column of G is B_N scaled by $B_N(0)$.

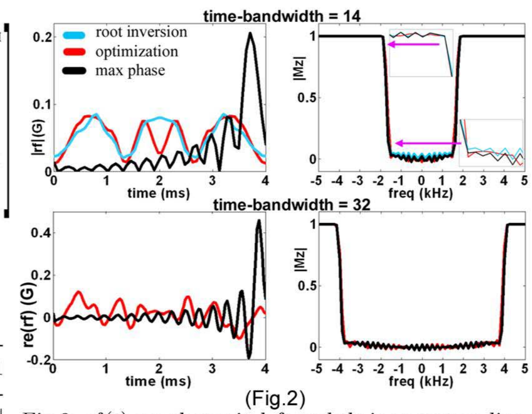

Fig.2 $rf(t)$ are shown in left and their corresponding $|M_z|$ are shown in right. Top row corresponds to $tb = 14$. Bottom row represents $tb = 32$.

$$
\begin{array}{ll}
\text{minimize} & \|B_N\|_\infty \\
\text{subject to} & 1-\delta_1 \le |H(\omega)| \le 1+\delta_1\,, \quad \omega \in [0, \omega_p] \\
 & |H(\omega)| \le \delta_2\,, \qquad \omega \in [\omega_s\,,\, \pi]
\end{array}
$$

Eq.1

$$
\begin{array}{ll}
\text{minimize}_{G\,,\,r} & \|\mathrm{diag}(G)\|_\infty \\
\text{subject to} & R(\omega) = r_{\mathrm{re}}(0) + 2\sum_{n=1}^{N-1} \mathrm{re}\{r(n)e^{-j\omega n}\} \\
 & (1-\delta_1)^2 \le R(\omega) \le (1+\delta_1)^2, \quad \omega \in [-\omega_p\,,\, \omega_p] \\
 & R(\omega) \le \delta_2^2\,, \qquad \pm\omega \in [\omega_s\,,\, \pi] \\
 & R(\omega) \ge 0\,, \qquad \omega \in [-\pi\,,\, \pi] \\
 & r(n) = \mathrm{trace}(I_n^{\mathrm{T}} G)\,, \qquad n = 0 \ldots N-1 \\
 & G \succeq 0 \\
 & \mathrm{rank}(G) = 1
\end{array}
$$

Eq.2

Excepting the rank constraint, this problem is convex; this means, it is numerically solvable for its global minimum by prevalent and readily available software programs. The rank constraint is met via method of *convex iteration.* Polynomial B_N is taken from the first column of G. Having B_N, the RF waveform $rf(t)$ is found by SLR transform. We use our optimization to design a saturation pulse with $tb = 14$ and then compare peak RF amplitude with those from a maximum-phase design with and without combinatorial search of all root inversion. We show another example of a large time-bandwidth design ($tb = 32$) for which it is not practical to obtain minimum peak RF amplitude by root inversion.

Results Figure 2 (left) shows RF waveforms $rf(t)$ from the optimization (red), and maximum-phase design with (blue) and without (black) root inversion. The design parameters are: $tb = 14$ (pulse duration $= 4ms$), $\delta_1 = 2\%$, $\delta_2 = 0.1\%$. Figure 2 (right) shows their corresponding $|M_z|$, where $|M_z| = 1 - 2|\beta|^2$ and β is the Cayley-Klein parameter obtained from SLR transform. All three designs meet specifications, while the optimization technique has lowest peak RF amplitude.

Discussion We presented a method to design an RF saturation pulse with globally minimum peak RF power without the need for exhaustive root-inversion search. For saturation RF pulses ($rf(t) \approx B_N$), minimum peak $rf(t)$ can be obtained by Convex Optimization for which the objective is to find a B_N having the lowest possible peak amplitude while simultaneously satisfying the given filter design parameters. Instead of working directly with impulse response, our technique employs its autocorrelation and imposes a rank constraint on the autocorrelation matrix G. The main diagonal of G is $[B_N(0)B_N(0)^{\mathrm{H}},\, B_N(1)B_N(1)^{\mathrm{H}}, \cdots]^{\mathrm{T}}$. Minimizing $\|B_N\|_\infty$ is therefore equivalent to minimizing $\|\mathrm{diag}(G)\|_\infty$. Once the program has achieved global minimum, B_N can be extracted from the first column of G. Columns of autocorrelation matrix G are simply copies of B_N having different scale factors; consequently, $\mathrm{rank}(G) = 1$. But a rank constraint is not convex. We overcome this by using the method of convex iteration. The number of iterations varies but total computation time is usually on the order of minutes in Matlab. Since RF power is directly related to SAR safety limits, this method is most relevant in high field applications and large time-bandwidth-product designs. Possible applications include TOF (selective arterial or venous imaging), outer volume suppression pulses, spectral saturation/supression.

Reference 1.Tuan-Khanh *et.al.* MRM,43(1),2000. 2.Merideth *et.al.* HBM,30(10),2000. 3.Josan *et.al.* MRM,61(5),2009. 4.Shinnar *et.al.* MRM,12(1),1989. 5.LeRoux *et.al.* JMR,8,1988. 6.Pauly *et.al.* IEEE Trans Med Imaging,10,1991. 7.Schulte *et.al.* JMR,166(1),2004. 8.Wu *et.al.* Applied and Computational Control,vol.1.Birkhuser Boston,1999.