## Nonuniform and multidimensional Shinnar-Le Roux RF pulse design

W. A. Grissom<sup>1</sup>, G. C. McKinnon<sup>2</sup>, and M. W. Vogel<sup>1</sup>

<sup>1</sup>GE Global Research, Munich, Bavaria, Germany, <sup>2</sup>GE Applied Science Lab, GE Healthcare, Milwaukee, Wisconsion, United States

## **Introduction**

The Shinnar-Le Roux (SLR) RF pulse design algorithm [1,2] is the most widely-used method for designing one-dimensional RF pulses on constant gradient waveforms, due to its intuitiveness and optimality. However, the SLR algorithm is presently limited in that it is only capable of designing one-dimensional pulses played simultaneously with constant gradient trajectories. In this work a new nonuniform SLR (NUSLR) RF pulse design framework is presented that extends the capabilities of one-dimensional SLR pulse design to multiple dimensions and time-varying gradient waveforms. The method is demonstrated in large-tip spectral-spatial and spiral pulse designs.

#### Theory

Two fundamental problems prevent the direct application of the classical SLR algorithm to pulse design on nonuniform and multidimensional gradient trajectories. The first is that a nonuniform trajectory of  $N_t$  samples generates  $\alpha$  and  $\beta$  polynomials with up to  $2^{\text{Nt-1}}$  terms that do not generally simplify. Because SLR seeks to design the coefficients of polynomial terms, the problem becomes computationally intractable. To address this issue, a novel problem parameterization was developed: the polynomials were cast in terms of  $N_t+1$  new variables  $a_0$  and p, defined as:

$$a_0 = \prod_{j=1}^{N_t} C_j$$
  $p_j = S_j/C_j$ 

 $a_0 = \prod_{j=1}^{N_t} C_j \quad p_j = S_j/C_j,$  where  $C_j$  and  $S_j$  are the cosine and sine of the half-angle excited by the pulse at time point *i* [2].

The second problem faced in extending SLR to multiple dimensions is the specification of the  $\alpha$  polynomial's phase pattern. This issue was obviated in the nonuniform SLR (NUSLR) method by designing on the  $\beta$  polynomial only, while maximizing  $a_0$  in order to minimize RF power and simultaneously enforcing a rotation magnitude constraint that ensures the solution  $a_0$  and p correspond to a realizable pulse.

## **Design Examples**

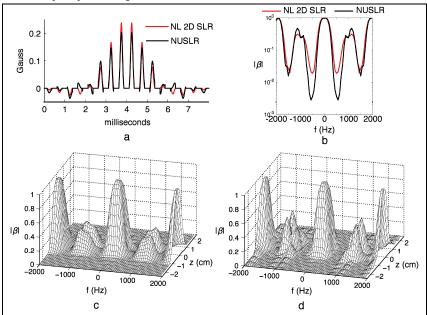
The NUSLR algorithm was implemented and compared to the nonlinear (NL) 2D SLR method for large-tip spectral-spatial pulse design [3], and compared to the linear class small excitation-approximation method for large-tip spiral pulse design [4]. Figure 1 shows that the method produces a pulse with significantly lower fat stop band excitation than the approximate NL 2D SLR method, while Fig. 2 shows that the

method produces a spiral in-out pulse with significantly lower stop band excitation than the small excitation-approximation pulse.

# Conclusion

A new method for multidimensional Shinnar-Le Roux RF pulse design was introduced that comprises a novel parameterization of the design problem, as well as novel optimization strategies to design multidimensional RF pulses. The method extends the ease of use and optimality provided by the classical Shinnar-Le Roux algorithm to multiple dimensions and nonuniform gradient trajectories. The method demonstrated to provide improvements in accuracy when compared to approximation-based methods for large-tipangle spiral and spectral-spatial pulse design.

References [1] M Shinnar et al. MRM 12:81-87, 1989. [2] JM Pauly et al. IEEE TMI, 10:53-65, 1991. [3] JM Pauly et al. MRM 29:776-82, 1993. [4] JM Pauly et al. JMR 82:571-87, 1989.



*Figure 1: Large-tip spectral-spatial pulse* 8 ms Fat-suppressing spectral-spatial  $\pi$  pulses were designed for 3T with a bipolar z gradient using the conventional NL 2D SLR method and the NUSLR method. Both methods used the same target  $\beta$  profile and error weighting. The NL 2D SLR method assumes a rectilinear gradient trajectory; the resulting waveform (a) comprises a train of nearly identical subpulses, while the NUSLR subpulses are strongly modulated, reflecting the nonuniform sampling of excitation k-space. (b) A maximum intensity projection in the slice dimension shows that the NUSLR method achieves almost an order of magnitude reduction in the 460 Hz fat excitation, which is also visible in the  $\beta$  profiles in (c) and (d).

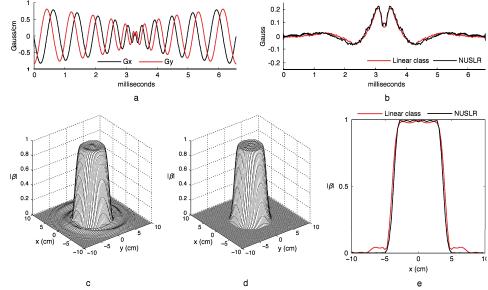


Figure 2: Large-tip spiral in-out pulse Equiripple spiral in-out pulses were designed using the linear class and nonuniform SLR methods. (a) Spiral in-out gradient waveforms. (b) The designed pulses. (c) The pattern excited by the linear class pulse is slightly wider than that of the NUSLR pulse (d) and contains large stopband excitation errors (maximum error 0.0443) that are also visible in their y=0 profiles (e) due to large gaps between successive spiral turns, while the NUSLR pulse excites a pattern with a maximum stopband error of 0.0044.