Accelerating Parallel Acquisition Reconstruction with Sparse Matrix Transformations

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INTRODUCTION

Parallel imaging techniques increase the speed of acquiring data during MRI. These techniques accelerate collection of frequency information by intentionally undersampling in *k*-space to obtain aliased images from an array of receive coils. Spatial sensitivity information for each coil is used to reconstruct data in either *k*-space or the image domain. Increasing numbers of receive coils and higher acquisition acceleration factors add computational complexity to the reconstruction of images acquired from the scan, with high orders of acceleration producing lengthy post-acquisition reconstruction times. As receive arrays become progressively larger, and higher acceleration factors more desirable, the post-acquisition reconstruction delays can become a limiting factor, negating some of the benefits of faster acquisition.

SENSE [1] provides a general framework for the reconstruction process in the image domain. SENSE inverts coil sensitivity data to “unfold” and combine the aliased image, with such operation dependent on Fourier transform techniques that utilize complex multiplications.

The sparse matrix transform (SMT) generalizes the fast Fourier transform (FFT), replacing the “butterflies” of the FFT with a series of Givens rotations [2]. Sparse representations of the unfolding matrices can be formed using an approximation of their singular value decomposition (SVD) generated by the SMT. The computational complexity of reconstruction can be reduced using these sparse representations of unfolding matrices with minimal effects on final image quality.

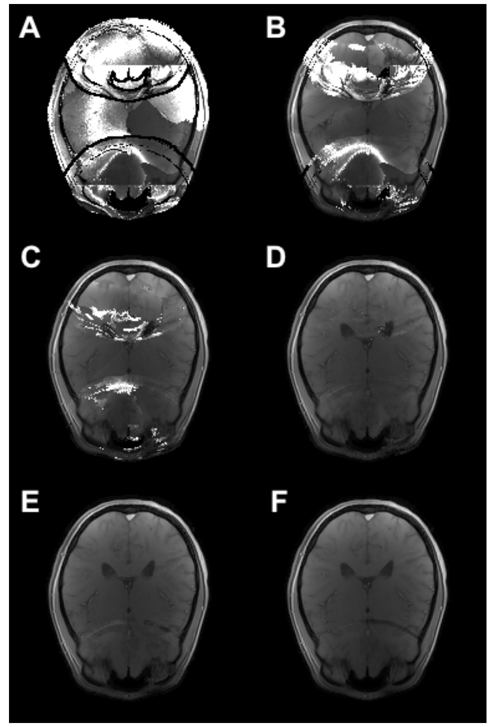

Fig 1. Sparse unfolding matrix reconstructions of a single parallel-acquired image (R=2). Approximate SVD matrices were found using (A) *k* = 1, (B) 2, (C) 4, (D) 8, and (E) 16 Givens rotations. Compare to (F) original reconstruction using traditional SENSE.

METHODS

Prototyped in MATLAB, the incorporation of SMT into SENSE involves approximating the unfolding matrix, *U*. In the standard implementation of SENSE, this matrix is calculated for each point in the aliased receiver image. Here the SMT is used to approximate the singular value decomposition of the unfolding matrix as $U = W S T^H$. This approximation is found iteratively, setting $S_0 = U$ and the initial rotation matrices W_0 and T_0 to identity. At the *n*-th iteration a pair of rotation matrices W_n and T_n is chosen to rotate the highest-cost off-diagonal pair to the diagonal:

$$(W_n, T_n) = \operatorname{argmax} \left\| diag\left(W^T S_{n-1} T\right) \right\|$$

with $S_n = W_n^T S_{n-1} T_n$. After *k* iterations, the final rotation matrices are the product over all iterations, $W = W_1 \cdots W_k$ and $T = T_1 \cdots T_k$, and further sparseness is introduced to the matrix S_k by discarding all off-diagonal elements, yielding $\underline{\hat{U}}$, a sparse approximation of *U*.

To evaluate this procedure, volumetric data (FSPGR) were acquired from a single volunteer using SENSE with R=2, with all data obtained on a 3T GE Signa HDx (Purdue MRI Facility) using a 16-channel Nova Medical receive array. The sparse representation of the unfolding matrix, $\underline{\hat{U}}$, was derived with *k* = 1, 2, 4, 8 and 16. Reduction in complexity was calculated at each *k* as the percentage decrease in the number of non-zero, non-unity multiplies from *U* to $\underline{\hat{U}}$. A second metric was assessment of the amount of time required to perform 100 reconstructions with *k*=8. Error was calculated as percent RMSE relative to reconstruction with non-sparse *U*.

RESULTS

Fig 1 presents images resulting from *k* = 1, 2, 4, 8 and 16, in addition to the original SENSE reconstruction (R=2). Fig 2 plots the percent RMSE against the reduction in multiplies achieved by SMT approximation. For 100 volume reconstructions, the sparse unfolding matrix reduced the time required for reconstruction by a factor greater than six.

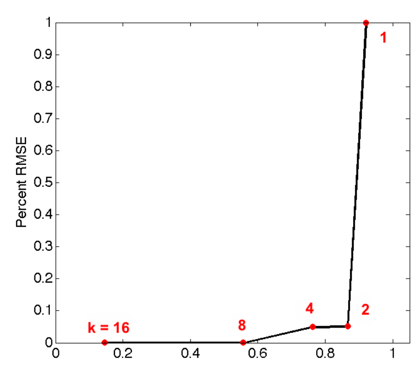

Fig 2. RMSE error vs. reduction factor for multiplies as function of iteration count, *k*.

DISCUSSION

The sparse representation of the SENSE unfolding matrix effectively reduces the computational complexity of the unfolding process without introducing appreciable error. After eight iterations of the approximation technique, the unfolding matrix can be represented using less than half of the original multiplies with little error. The nature of the decomposed unfolding matrix allows for reduced complexity in storing the required matrices, permitting more rapid reconstruction of repeated acquisitions of images acquired using parallel imaging techniques (e.g., CINE, fMRI).

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REFERENCES

[1] Pruessmann et al, *Magn. Res. Med.*, 1999; [2] Cao et al, *Adv. in Neural Info. Proc. Sys.*, 2008