Combination of arbitrary gradient encoding fields using SPACE RIP for reconstruction (COGNAC)

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Introduction: For parallel imaging it has recently been suggested to use non-linear radially symmetric gradients in order to better complement to receiving coil arrays [1]. Furthermore it has been assumed that non-linear gradients can reduce peripheral nerve stimulations and therefore allow faster gradient switching [2]. For example, using a radially symmetric gradient for frequency encoding, the object is projected onto concentric rings. For the encoding of the angular direction, a method is presented that uses both conventional gradients and the spatial information of the underlying receiver coil array. As the conventional k-space formalism needs gradients with uniform directions and therefore limits the possibilities of sampling, SPACE RIP [3] is used for reconstruction. In SPACE RIP an encoding matrix describes both, coil sensitivity profiles and gradient encoding. Reconstruction is done by inverting the encoding matrix. **C**ombination **O**f arbitrary **G**radient e**N**coding fields using SP**AC**E RIP for reconstruction (COGNAC) makes use of this approach to allow the use of arbitrary phase encoding fields for each phase encoding step.

Theory: The signal equation for one phase encoding step in COGNAC is the generalized form of the SENSE [4] equation, where all signals on a frequency isocontour folds onto one pixel. Furthermore the equation is supplemented with the phase induced by the phase encoding field and yields in matrixform

$$\vec{S} = \mathbf{C} \cdot \mathbf{G} \cdot \vec{\rho} = \mathbf{C'} \cdot \vec{\rho}$$

where $\vec{S}$ is the signal of the n_c receiver coils in the folded pixel, $\mathbf{C}$ denotes the coil sensitivity matrix and $\mathbf{G}$ the phase induced by the phase gradient. $\vec{\rho}$ represents the magnetization. If the encoding matrices of all n_c phase encoding steps are combined, an encoding matrix $\mathbf{C}'$ with $n_c \times n_p$ virtual coils is derived. The regularized reconstruction equation

$$\vec{\rho} = \left(\mathbf{C}'^H \mathbf{\Psi}^{-1} \mathbf{C}' + \lambda^2 \mathbf{I}\right)^{-1} \mathbf{C}'^H \mathbf{\Psi}^{-1} \vec{S}'$$

follows out of the Tikhonov cost function. Here H indicates the transposed and complex conjugate. $\mathbf{\Psi}$ is the noise covariance matrix, λ the regularization parameter, $\mathbf{I}$ the identity matrix and $\vec{S}'$ contains the signal of all virtual coils in the folded pixel.

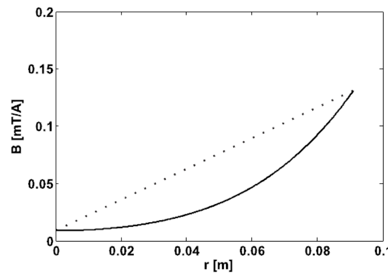

Figure 1: Magnetic field of the home-made non-linear (continuous) and a hypothetical linear (dotted) radially symmetric gradient. The latter is non-physical as the derivation of the magnetic field is not continuous at the origin.

Methods: For a proof of principle, COGNAC simulations were performed with a hypothetical linear radially symmetric gradient (Figure 1) for frequency encoding and a conventional linear phase gradient in left-right direction (x). This leads to data in the k_r-k_x-domain. The coordinate k_r is obtained after inverse Fourier-transformation (IFT) in the radial (r) dimension. Thereafter each concentric ring can be reconstructed separately. This is done in polar coordinates. Each point on the k_x axis can be understood as a projection of the object, modulated by the sensitivity maps and the phase induced by the phase gradient, thus representing the signal of a virtual coil. The encoding is described by the rows of **C'**. With the signal of all virtual coils, the image can be unfolded in the angular direction. After reconstruction of all rings, the final image in polar coordinates is derived. Coordinate transformation leads to the final image (Figure 3).

In a second simulation the phase gradient rotates at constant magnitude by an angle of $2\pi / n_p$ per phase encoding step. The magnitude is chosen to correspond to the original Cartesian resolution. It is described by the outmost point of Cartesian k-space ($k = \pi$).

The matrix size of the original image is 256×256 . Both simulations were performed with 256 ($R = 1$) and 128 phase encoding steps ($R = 2$). The first simulation was verified by measurements with a 1.5T clinical scanner. Cartesian resolution is here 128×128 . The magnetic field of the homemade radial gradient is shown in Figure 1. Non-linearity of the radially symmetric gradient is dealt with by interpolation after the IFT. Measurements were acquired with a 12-channel head coil. Simulations were performed with sensitivity maps from the same coil.

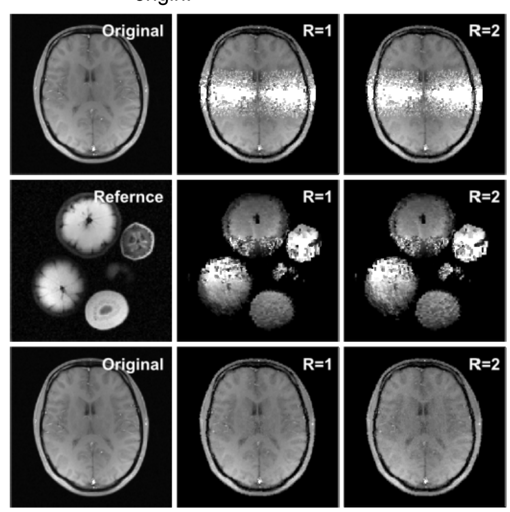

Figure 2: COGNAC reconstructions with radial frequency encoding. Top: Simulations with phase encoding in left-right direction. Middle: Measurements with phase encoding in left-right direction. Bottom: Simulations with a rotating phase gradient.

Results and Discussion: Figure 2 shows the results of the COGNAC simulations. With phase encoding in the left-right direction it can be seen, that reasonable reconstruction is only possible, where phase and frequency gradients are not parallel to each other. Gradient encoding here is mirror-symmetrical to the line, where the gradients are parallel to each other. Therefore two aliased pixel can only be separated by coil sensitivities. This leads to a dependency of the reconstruction properties on the distance to the symmetry axis.

The simulation with the rotating phase gradient shows a more homogeneous reconstruction at the cost of slightly higher noise amplification. This can be understood similar to the intrinsic undersampling of the outer k-space by standard radial acquisition. However, an exact analysis of resolution, SNR and artifacts is beyond the purpose of this paper.

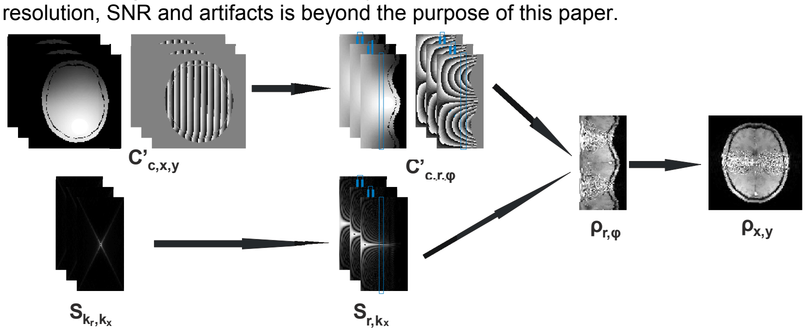

Figure 3: Reconstruction workflow using the example of phase encoding in left-right direction. Coil maps are shown as magnitude (left) and phase images (right). For explanation see text.

The measurements show good consistency with the simulations. Poor SNR is due to hardware setup. Due to the non-linearity of the radially symmetric gradient, the resolution depends on r.

In conclusion, taking advantage of the SPACE RIP reconstruction technique, COGNAC allows to combine any frequency gradient (here r-gradient) with any phase encoding. Therefore it can be used to better adapt sampling to the coil configuration. As a potential application, the formalism can be extended by temporal frequency filtering similar to k-t-BLAST [5] in order to allow dynamic imaging with very high frame rates.

References: [1] Stockmann J., et al., MRM, 2010, 64:447–456; [2] Hennig J., et al., MAGMA, 2008, 21:5-14; [3] Kyrakos WE., et al., MRM, 2000, 44:301-308; [4] Pruessmann K., et al., MRM, 1999, 42:952-962; [5] Tsao J, et al., MRM, 2003, 50:1031-1042.