Fast image reconstruction in the presence of dynamic higher-order fields

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Introduction: MRI gradient systems are conventionally designed to produce spatially linear B_z fields. Such linear encoding fields allow for uniform resolution over the entire FOV and enable the use of the Fast Fourier Transform (FFT) for efficient image reconstruction. However, despite continuous efforts, modern MR gradient systems still produce dynamic spatially higher-order fields, stemming from Eddy currents and Maxwell terms. Neglecting these field components causes image artifacts such as image distortion and aliasing. Recently it has been proposed to incorporate arbitrary dynamic higher order-field perturbations into image reconstruction and remove related distortions [1]. The reconstruction times for larger matrices were shown to be on the order of minutes. In this work the reconstruction time is massively reduced by adopting the idea of conjugate phase reconstruction[2], exploiting the fact that the spatio-temporal variation of these higher-order fields is of low order.

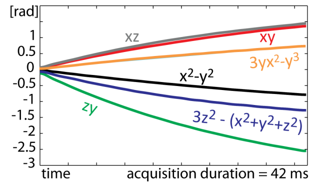

Fig1: Phase effects of higher-order fields within the imaging volume during a DW-EPI acquisition

Theory: In the presence of higher-order fields, spatial encoding can be written as $E(t,\boldsymbol{r}) = e^{-i\boldsymbol{k}(t)\boldsymbol{r}}e^{-i\varphi(\boldsymbol{r},t)}$ (1) where $\boldsymbol{r}$ denotes the spatial coordinate, $\boldsymbol{k}$ the k-space coordinate and φ represents the spatio-temporal phase term induced by the higher-order fields. An SNR optimal image is reconstructed by solving $\boldsymbol{\rho} = (\boldsymbol{E}^H\boldsymbol{\Psi}^{-1}\boldsymbol{E})^{-1}\boldsymbol{E}^H\boldsymbol{\Psi}^{-1}\boldsymbol{\sigma}$, where $\boldsymbol{\rho}$ is the unknown vector of object pixels, $\boldsymbol{\sigma}$ is the acquired signal, and $\boldsymbol{\Psi}$ is the noise covariance[3]. The encoding matrix $\boldsymbol{E}$ is defined according to Eq.1 for all discrete time points and pixels in the image. Finding a solution of Eq. 2 is favorably performed by iterative solvers, which involve the multiplication of large matrices $\boldsymbol{E}$ and $\boldsymbol{E}^H$ with vector data $\boldsymbol{x}$:

$$(\boldsymbol{Ex})_\kappa = \sum_{\lambda}^{R} e^{-i\boldsymbol{k}(t_\kappa)\boldsymbol{r}_\lambda}e^{-i\varphi(\boldsymbol{r}_\lambda,t_\kappa)}x_\lambda \quad (2.1) \qquad \text{and} \qquad (\boldsymbol{E}^H\boldsymbol{x})_\lambda = \sum_{\kappa}^{T} e^{-i\boldsymbol{k}(t_\kappa)\boldsymbol{r}_\lambda}e^{-i\varphi(\boldsymbol{r}_\lambda,t_\kappa)}x_\kappa \quad (2.2),$$

where λ and κ count R pixels and T k-space positions respectively. Following the idea of CPR [2], the higher-order signal modulation $e^{i\varphi(r,t)}$ is expanded in a sum of functions of separated spatial and temporal dependence

$$e^{-i\varphi(r,t)} \approx \sum_{l=1}^{L} c_l(t)\, b_l(r) \quad (3),$$

where L denotes the number of basis functions b_l and corresponding coefficients c_l. By inserting Eq. 3 into Eqs.2, a fast approximate multiplication of $\boldsymbol{E}$ and $\boldsymbol{E}^H$ can be achieved at the complexity of only L FFT's:

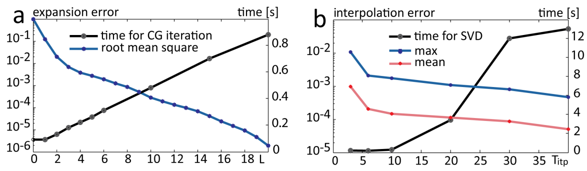

Fig2: Error and computation times relating to the size of SVD expansion ($\boldsymbol{L}$) (left) and to the interpolation of the numerical basis/coefficients ($\boldsymbol{b}/\boldsymbol{c}$)(right)

$$(\boldsymbol{Ex})_\kappa = \sum_{l=1}^{L} c_l(t_\kappa)\sum_{\lambda}^{R} b_l(\boldsymbol{r}_\lambda)\, e^{-i\boldsymbol{k}(t_\kappa)\boldsymbol{r}_\lambda}x_\lambda = \sum_{l=1}^{L} c_l(t_\kappa)\; IFFT\{b_l(\boldsymbol{r}_\lambda)x_\lambda\} \quad (4.1) \qquad \text{and analogously} \qquad (\boldsymbol{E}^H\boldsymbol{x})_\lambda = \sum_{l=1}^{L} b_l^*(\boldsymbol{r}_\lambda)\; FFT\{c_l^*(t_\kappa)x_\lambda\} \quad (4.2).$$

To keep L small, a compact basis has to be chosen. For higher-order fields of low spatio-temporal variation, it is possible to calculate a compact basis of $\boldsymbol{E}$ by Singular Value Decomposition: $\boldsymbol{USV} = SVD(\boldsymbol{E})$. Here $\boldsymbol{U}$ and $\boldsymbol{V}$ are matrices holding the left and right singular vectors and $\boldsymbol{S}$ the singular values. As $\boldsymbol{E} \approx \boldsymbol{U'S'V'} := (\boldsymbol{U}_1,..,\boldsymbol{U}_L)(\boldsymbol{S}_1,..,\boldsymbol{S}_L)(\boldsymbol{V}_1,..,\boldsymbol{V}_L)^T$, one can choose $b_l = \boldsymbol{U'}$ and $c_l = \boldsymbol{S'V'}$. For efficiency the SVD is performed on a lower resolution (T_{itp}, R_{itp}). Subsequently b_l and c_l are interpolated to the desired resolution (T, R). The accuracy of the approximation is determined by the choice of L, T_{itp} and R_{itp}.

Methods: The method is demonstrated for the case of diffusion weighted imaging (DWI), where strong higher-order terms occur (Fig. 2). Data was acquired with an 8-element head coil-array during typical SS-DW-EPI experiments (b = 1000s/mm^2, readout duration = 45ms) with (matrix = 144^2), and without SENSE (R = 3 /matrix = 78^2). The field evolution including higher-order fields (Fig. 1) was recorded by concurrent field monitoring [4]. To evaluate the accuracy of the method, MR data of a Shepp-Logan phantom was simulated by Eq. 1 (direct matrix multiplication) on the basis of this field information. Images were reconstructed (AMD Opteron, 2.6 GHz, 8GB RAM) using the presented method. The interpolation error and calculation time for the SVD was evaluated for different numbers of supporting points T_{itp}. The SVD expansion error was directly obtained from the singular values. For demonstration purposes the phantom was also reconstructed without accounting for DW induced fields. To validate the method, the in-vivo data was reconstructed with the proposed method as well as by regular matrix multiplication[1].

Results: Fig2a shows an SVD error exponentially decreasing with L, while the time for one CG iteration step increases linearly. Even for large L, the proposed method allows for shorte ring the time of one iteration from 79s (reference method) to below a second. As expected, the interpolation error (Fig2b) decreases with increasing T_{itp}. The calculation time for the SVD increases non-linearly with T_{itp}. The phantom images (Fig 3a-c) show that distortions are effectively eliminated (Fig 3c) using a small basis ($L = 6$ and $T_{itp} = 10, R_{itp} = R$). Similarly, the differences (Fig 3e) to the reference method (Fig 3d) stay within the expected error bounds

Discussion and Conclusion: A drastically accelerated higher-order image reconstruction was introduced, utilizing the computational efficiency of the FFT. In this work the use of a custom numerical basis set is proposed, calculated by SVD. It is optimal in terms of minimizing the number of basis functions for a given root-mean-square error on the approximation of $\boldsymbol{E}$. Other basis functions may be used, catering to other optimization criteria, such as fast computation of $\boldsymbol{b}$ and $\boldsymbol{c}$ or other interpolation properties. Further acceleration of the algorithm might be achieved by also interpolating in the spatial domain.
The achieved speed-up admits higher-order reconstructions lasting on the order of seconds. MR systems with known higher-order response or equipped with field monitoring capability can thereby reconstruct faithful images at clinically acceptable reconstruction times.

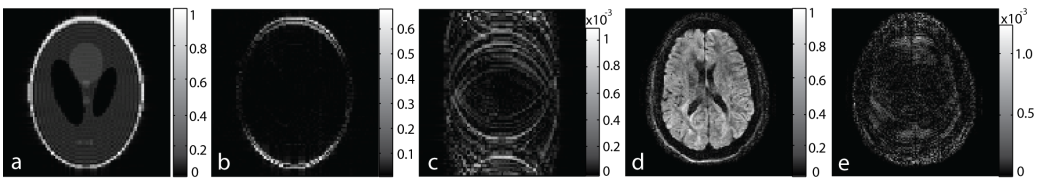

Fig 3: EPI data simulated in the presence of DW induced (higher-order) fields (a-c). Reconstructed with full non-interpolated SVD basis (a); neglecting DW induced fields (*L=0*) (b) and with the proposed method (*L=6*, $\boldsymbol{M}_{itp} = \mathbf{10}$)(c). d:DW image reconstructed with the reference method (full matrix multiplication); e: difference to the proposed method.

References: :Wilm et al. ISMRM 2009, 562; 2: Maeda et al. IEEE Trans. Med. Imag. 7, 26-31;
3: Pruessmann et al, MRM46:638
4: Barmet et al. ISMRM 2010, 216