Fast Image Reconstruction for Generalized Projection Imaging

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Introduction: Recent work has demonstrated the concept of MRI spatial encoding with more flexible gradient systems [1], potentially leading to improved encoding efficiency and imaging speed. The main idea is to use several magnetic fields of different geometric shapes [1,2] for encoding. It could be shown that for two arbitrary encoding fields images can be reconstructed robustly and fast [3]. However, when more than two fields are employed, reconstruction time becomes a serious issue. The main problem is the dense structure of the large encoding matrix. For pure frequency-encoding, it has been proposed that a sparser structure might result using a Fourier-domain representation of the signal equation [4]. It also has been correctly observed that the box-shape of the acquisition window has to be considered to get consistent reconstructions. However, no consistent reconstruction was formulated and therefore high-quality image reconstructions were based on the dense time-domain forward model.
The purpose of this abstract is to present more insight into this interesting topic and present consistent faster reconstructions based on a sparse Fourier-domain representation.

Theory: Consider arbitrarily shaped, however constant encoding fields $B_j(\vec{x})$ during readout (example fields are shown in Fig. 1). The signal of rf-receive coil α and projection j then has the following form:

$$(1) \qquad s_\alpha(t,j) = w(t)\int \rho(\vec{x})c_\alpha(\vec{x})e^{i\phi_j(\vec{x})}e^{it\gamma B_j(\vec{x})}d\vec{x}$$

The phase $\phi_j(\vec{x})$ is due to phase-encoding and the width of the box-shaped window $w(t)$ corresponds to the duration of signal recording (top of Fig. 2a). The equivalent frequency-(Fourier)-domain representation of the signal is then given by

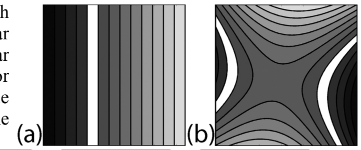
$$(2) \qquad s_\alpha(\omega,j) = \int \rho(\vec{x})c_\alpha(\vec{x})e^{i\phi_j(\vec{x})}\widehat{w}(\omega-\gamma B_j(\vec{x}))d\vec{x}$$

For reconstruction the cost function $\|s-E\rho\|$ is minimized with vectorized signal and image and encoding matrix E. The discrete Fourier Transform (DFT) is unitary and therefore an equivalent reconstruction results when formulated in frequency domain: $\|s-E\rho\| = \|DFT\ s - DFT\ E\rho\|$. The transformed encoding matrix $\tilde{E} = DFT\ E$ can be interpreted as a projection operator; it can be taken directly from Eq. (2). A fast reconstruction would result by simply back-projecting along the isocontours of the encoding field (cf. Fig. 1). This would correspond to a Fourier-domain window $\widehat{w}(\cdot)$ of a delta function or a box-shaped function (bottom of Fig. 2c,d). However, the correct window is a sinc-function (see bottom of Fig. 2a) and therefore each point must be back-projected not only along one line, but onto the complete image. This implies a dense encoding matrix $\tilde{E}$. Reconstruction is feasible using this method, but there is no improvement in computation time compared to the time-domain reconstruction: The two methods give exactly the same results. Thresholding the side-lobes of the sinc sparsifies the encoding matrix, but this approach results in inconsistent reconstruction. A fast *and* consistent reconstruction results when the signal data are filtered with a window which suppresses the side-lobes of the sinc (Fig. 2b).

Methods: Simulations were performed with MATLAB (The Mathworks, Natick, USA) based on a Cartesian trajectory with quadrupolar PatLoc encoding. Measurements were performed on phantom data with a 4D-RIO trajectory [2] applied to combined linear and quadrupolar field encoding and in vivo measurements were based on pure quadrupolar PatLoc encoding with a twofold undersampled radial trajectory.

Results: Reconstruction results from simulations are shown in Fig. 3. The comparison of Fig. 3a with 3b confirms the equivalency of time-domain and frequency-domain reconstruction. A consistent, but in this case 40-fold faster reconstruction results when the signal data are filtered with a Kaiser-Bessel filter. Reconstruction fails, when the windowing effect is only considered in the encoding matrix and not also in the acquired signal data. Reconstruction is also inconsistent when the very sparse and fast reconstruction based on a box-shaped frequency window is used. The measurements shown in Fig. 4 confirm the results of the simulations.

Figure 1: Magnetic field strength during one example readout. (a) Linear gradient field. (b) Combination of linear and quadrupolar PatLoc field. For reconstruction in frequency domain the signal data are back-projected along the isocontour lines of the encoding field.



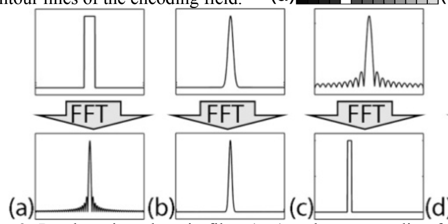


Figure 2: Readout time-domain filter (top) and corresponding frequency-domain filter (bottom). (a) The natural filter is box-shaped with a width corresponding to the time duration of the ADC. The corresponding frequency-filter is a sinc-function with non-local support. (b) With a Kaiser-Bessel filter, the frequency support is localized. (c) The simplest back-projection algorithm corresponds to a box-shaped frequency-domain support, which, however, is only consistent with a sinc-filter in time-domain, that is not useful to implement. (d) The perfect frequency-domain filter would be a delta-function, corresponding to an infinitely long readout.

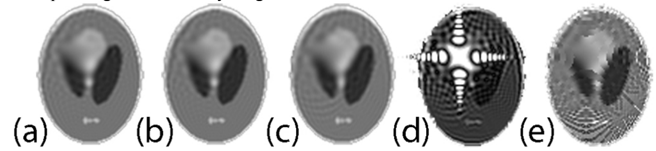


Figure 3: Reconstructed images for a Cartesian trajectory with quadrupolar PatLoc encoding. (a) Unfiltered time-domain and (b) frequency-domain reconstructions give equivalent results. (c) With a Kaiser-Bessel filter a consistent faster reconstruction results. (d) Not filtering the signal data leads to inconsistent reconstruction. (e) Back-projection along box-shaped bands does not correspond to a consistent reconstruction.

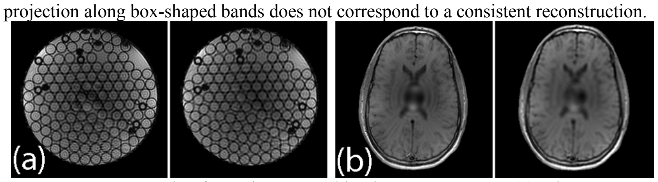


Figure 4: Reconstruction from phantom and in vivo measurements (a) using a 4D-RIO PatLoc trajectory. Left: unfiltered, slow reconstruction. Right: faster reconstruction and filtered with a Kaiser-Bessel window; (b) using a radial PatLoc trajectory with pure quadrupolar encoding. Left: unfiltered. Right: filtered. The filtered reconstruction produces consistent reconstructions with a minor reduction in resolution.

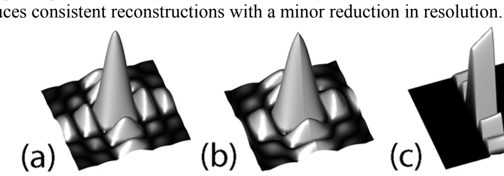


Figure 5: The central part of a PSF for three reconstruction methods for standard Cartesian Fourier imaging with multiple receiver coils: (a) unfiltered reconstruction (b) filtered with a Kaiser-Bessel window (c) back-projection along sharp frequency bands. Frequency encoding is from left to right. It can be observed that in (b) compared to (a) the resolution is not affected along the phase-encoding direction. However, along the frequency-encoding direction some resolution loss occurs. The PSF in (c) indicates that simple back-projection along bands cannot produce high-quality MR images.

Discussion: In this abstract we have shown that time-domain and frequency-domain reconstructions are fully equivalent when the encoding field does not change during readout (or at least is piecewise-constant as in 4D-RIO). This is the case for a large class of encoding trajectories. Whereas in time-domain the encoding matrix has highly oscillating exponentials in the temporal dimension, in frequency space only a weighting occurs corresponding to the DFT of the time window. Therefore, efficient methods can be applied in frequency-domain to get fast sparse reconstructions. This can be achieved by adequately filtering the data. Filtering does not affect resolution in the phase-encoding direction. However, a minor loss of resolution occurs along the direction of frequency encoding corresponding to the applied filter (see Fig. 5). We have also shown that the filter effect must be considered in both the acquired signal as well as in the construction of the encoding matrix to get consistent image reconstructions.

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References: [1] Hennig et al., MAGMA 21(1-2):5–14, 2008; [2] Gallichan et al., MRM 2010 (in press); [3] Schultz et al., MRM 64:1390-1403, 2010; [4] Stockmann et al., MRM 64:447-56, 2010.