

Introduction: Compressed Sensing(CS) and Parallel Imaging(PI) are fundamentally different methods, even though both of which aim to speed up the data acquisition. Recently the combination of CS and PI were proposed to achieve an even higher reduction in scanning time[1,2]. In this paper, we propose a novel image reconstruction method in which CS and PI are executed using only single set of signal. The signal obtained in phase-scrambling Fourier transform (PSFT) technique[3] can be described by the convolution integral of quadratic phase function and object function. Since the distribution of PSFT signal strongly reflects the distribution of the object, the application of a weighting function to the PSFT signals has a similar effect as the application of the weighting function to the object. Therefore SENSE reconstruction using a single signal is feasible by producing another folded image having another weighting function [4]. PSFT signal has also the feature that interpolation of the signal which violated the sampling theorem is executed successfully[5] since amplitudes of the adjacent samples of the PSFT signal are highly interrelated, which feature is suited for compressed sensing. Here, we propose a new imaging method which combine CS and PI using single signal to achieve higher reduction factor.

Method: Phase-scrambling Fourier transform (PSFT) imaging[3] is a technique whereby a quadratic field gradient is added to the pulse sequence of conventional FT imaging in synchronization with the field gradient for phase encoding. The signal obtained in PSFT is given by Eq. (1):

$$v(k_x, k_y) = \iint_{-\infty}^{\infty} \rho(x, y) e^{-j\gamma b \tau (x^2 + y^2)} e^{-j(k_x x + k_y y)} dx dy, \quad (1) \quad v(x', y') e^{-j\gamma b \tau (x'^2 + y'^2)} = \iint_{-\infty}^{\infty} \rho(x, y) e^{-j\gamma b \tau [(x'-x)^2 + (y'-y)^2]} dx dy. \quad (2)$$

where $\rho(x, y)$ represents the spin density distribution in the subject, γ is the gyromagnetic ratio, and b and τ are the coefficient and impressing time, respectively, of the quadratic field gradient. Equation (1) can be rewritten as the Fresnel transform equation, as shown in Eq. (2), by using the variable substitutions $x' = -k_x/2\gamma b \tau$ and $y' = -k_y/2\gamma b \tau$ [4]. Subject images can be obtained by applying the inverse Fourier transform (IFT) to the signal of Eq.(1). The right-hand of Eq.(2) is a convolution integral of $\rho(x, y)$ and $\exp[-j\gamma b \tau (x'^2 + y'^2)]$ which imply that the distribution of signal amplitude resemble with the distribution of the object. Applying a weighting function $S(k_x, k_y)$ to the PSFT signal $v(k_x, k_y)$ is approximated by $S(x, y)\rho(x, y)$ [4] which implies that another signal having different weighting function can be produced using $v(k_x, k_y)$ and $S(k_x, k_y)$ after data acquisition. Single signal parallel imaging is achieved by using two different folded images, one of which was obtained by adding a weighting function to the PSFT signal before IFT.

Figure 1 shows the schematic of proposed method. Fig.1(a) show the undersampled PSFT signal and signal after CS is shown in (b). The image I_1 is a folded image by applying IFT to the signal (b). The image I_2 shown in (e) is also folded image but is reconstructed using weight-multiplied signal $v^2 = Sv$. The weighting function S numerically applied to the acquired signal and the resultant weighting function provided on the folded images I_2 have a difference to be exact, however, both have a close distribution and therefore, pseudo SENSE reconstruction is feasible by approximating the weight on the image as $S(x, y)$ [4]. To add the weighting function on the whole images without folding effect, the range of convolution integral of Eq.(2) must be larger than the size of the subject.

Results and Discussion: The CS adopt FREBAS transform[6] as a sparsifying function and soft-thresholding in the FREBAS domain is taken to minimizing the L_1 norm in the FREBAS domain. In the simulation experiments, PSFT signal is calculated using the MR volunteer image data according to the Eq.(1). The condition of experiments are as follows; $\gamma b \tau = 1.23$ rad/cm, $\Delta x = \Delta y = 0.1$ cm, the sampling length for phase encoding direction in Eq.(2) after CS is set to the same as FOV of full-size image; $\Delta y' = 2\Delta y$. We used a sigmoid function as a weighting function $S(k_x, k_y)$. Figure 2(d) shows the amplitude of PSFT signal which resemble with subject distribution (a). Figures (b), (c) and (e), (d) show the results of reconstructed images by proposed method and standard CS[8] when the reduction factor is 15%, 25%, respectively. The results of application to the experimentally obtained PSFT signal using 0.2T MR scanner are shown in Fig.3. Figure 3(a) shows the initially reconstructed image by zero-fill FT using 20% of data and (b) shows the image after CS. Figures (c) and (d) show the reconstructed images using fully-scanned data and proposed method using the signal after CS. Even some perturbations are observed on the image, clear image was obtained in proposed method.

Conclusion: We apply CS to pseudo parallel imaging using a single signal to achieve a higher reduction factor. The CS algorithm employed in this work takes soft thresholding in the FREBAS multi-resolution image space. Pseudo parallel imaging uses the resemblance of amplitude distribution between PSFT signal and subject image. The results of simulation studies and experiments using 0.2 T MRI scanner show promising results. Future work will test on real data which has phase variations.

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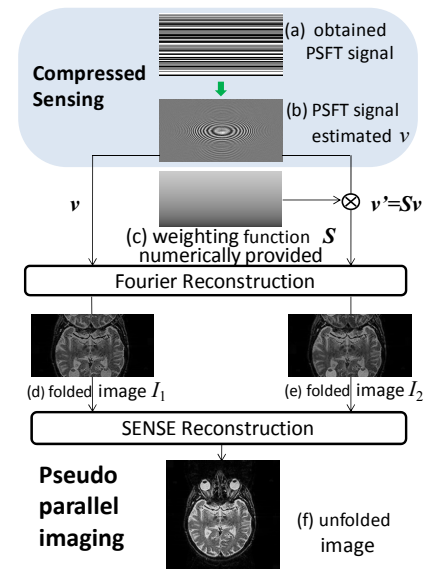


Fig. 1 Combination of compressed sensing and pseudo parallel imaging using a single signal.

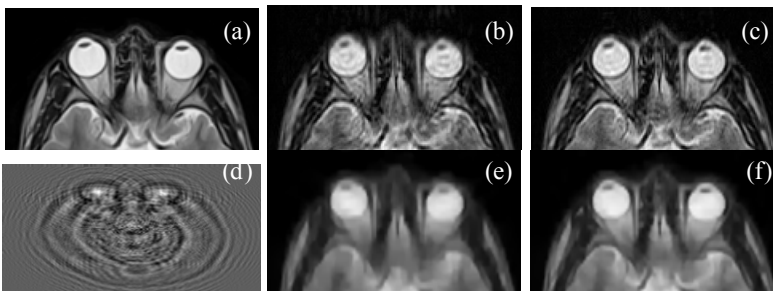


Fig. 2 Comparison of simulation experiments (a) gold standard, (b), (c) show images in proposed method using 15%, 25% of fully scanned data, (e) amplitude of PSFT signal after CS, (f), (g) and (h) are images by standard CS using 15%, 25% data.

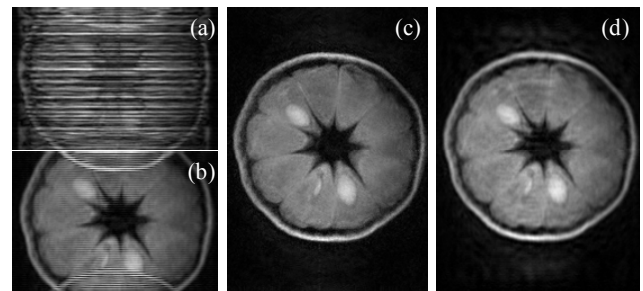


Fig. 3 Application to experimentally obtained PSFT signal. (a) initially reconstructed folded image, (b) folded image after CS, (c) fully scanned image, (d) 20% signal image in our method