# Compressed Sensing in Phase-encoded Multi-dimensional Magnetic Resonance Imaging

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**INTRODUCTION:** Recent advances in compressed sensing (CS) [1] have accelerated MRI by exploiting signal sparsity. However, the existed image recovery scheme [2] may be difficult to converge when the data matrix is large. The object of this study is to examine the applicability of CS to phase-encoded 3D MRI, which usually acquires a large data matrix with relatively long acquisition time. We propose an iterative reconstruction procedure that approximates the under-determined problem with a sequence of over-determined problems. We will demonstrate that this method is accurate, efficient and stable for recovering 3D MRI from half k-space acquisition.

**THEORY:** Our reconstruction scheme is based on the current CS-MRI reconstruction [2] that is solving an optimization problem (in Lagrange form) $\min\{\|M(FT(S))-b\|_2^2+\lambda_1\|S\|_1+\lambda_2 TV(S)\}$ where $S$ is image, $b$ is measured phase encoding line, the $FT$ is 3D Fourier transform, $TV$ is total variation and $M$ is undersampling operation. However, this optimization problem is difficult to solve when image size become large. Hence we replace the above optimization problem by a sequence of simplified optimization problems. As shown in Fig. 1, the new reconstruction scheme relies on the optimization problem $\min\{\|S-K\|_2^2+\lambda_1\|S\|_1+\lambda_2 TV(S)\}$, which is isolated with measurement $b$. We approximate the constraint $\|M(FT(S))-b\|_2^2$ by using an iteration loop (Fig. 1a) that iteratively impose the summation of $b$ (measured phase encoding line) and $S^*$(unmeasured phase encoding line) into $K$. The Lagrange function of inner optimization problem can be separated into two parts as $\{\|S-K\|_2^2+\lambda_1\|S\|_1+\lambda_2 TV(S)\}=\{\|M(FT(S))-b\|_2^2+\lambda_1\|S\|_1+\lambda_2 TV(S)\}+\|FT(S)-M(FT(S))-S^*\|_2^2$, the first part is the Lagrange function of the original optimization problem and the second part is an additional constraint that requires the optimum to be "localized". Hence proposed scheme generates a sequence of progressive locally optimal solutions to approximate the global optimum.

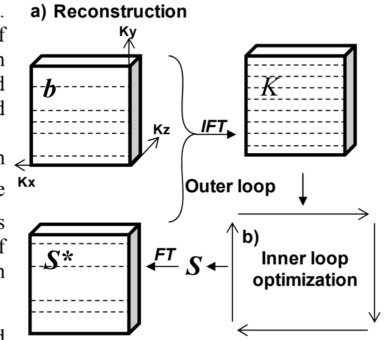


**Fig. 1:** Reconstruction scheme shows the 3D k-space data set: $b$ is the measurement; $S^*$ is solution of inner optimization; $K$ is the sum of $b$ and $S^*$. (**a**) The reconstruction scheme. (**b**) The inner loop optimization.

**METHODS:** the half k-space undersampling was achieved by randomly selecting the phase encode lines measured. The 3D acquisition order was: phase encoding in Kx-direction, phase encoding in Kz-direction and readout in Ky-direction. As shown in Fig. 3c, the sampling density function was quadratic with highest value in the center of Ky-Kz plane. The in-vivo rat brain experiment was performed in a 7T Bruker scanner. T1 contrast 3D high resolution image was acquired by a Modified Driven Equilibrium FT sequence [3] with TR/TE = 9/3 ms, Inversion Time = 1.2s, matrix size = 160×160×80, FOV = 32×32mm$^2$, slice thickness = 0.2mm, and NEX = 4. The 50% random sampling was performed retrospectively to the fully sampled dataset. The raw data was the zero filled to 256×256×80 before reconstruction.

**RESULTS:** In Fig.2, the value of Lagrange function of inner optimization is reduced after each inner loop and appears to approach convergence when the outer loop terminates in 20 setps. In Fig. 3b, the reconstruction results (50% k-space dataset) match up well with the original (100% k-space dataset). As shown the histograms of Fig. 3d, the error is less than 1/10 the signal of the subject. The error/image distribution is close to Gaussian with mean = -0.015 and standard deviation = 0.087 (in Fig. 3d, the background of image is removed by setting threshold at 3 times of noise level).

**DISCUSSION:** As illustrated in Fig.2, the reconstruction is stable because it is over-determined in every iteration step and each inner optimal solution is always close to the measurement in a least mean square sense. And it is efficient as no Fourier transform used in the inner loop. Fourier transform is expensive in terms of computation. The reconstruction is accurate, as illustrated in Fig. 3. It is important to know that, in the convex optimization, any locally optimal value is also globally optimal. Hence, the solution of inner optimization should converge to the global sparse solution. The L-1 norm may slightly "shrink" the recovered signal and enlarge the difference (error in Fig. 3.b).

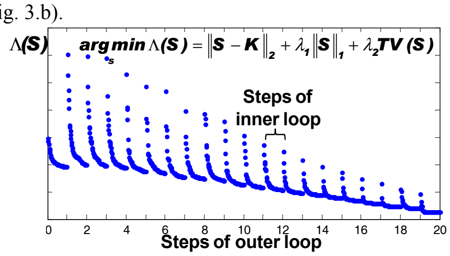


**Fig. 2:** Showing the proposed method minimize Lagrange function of inner optimization in each iteration step.

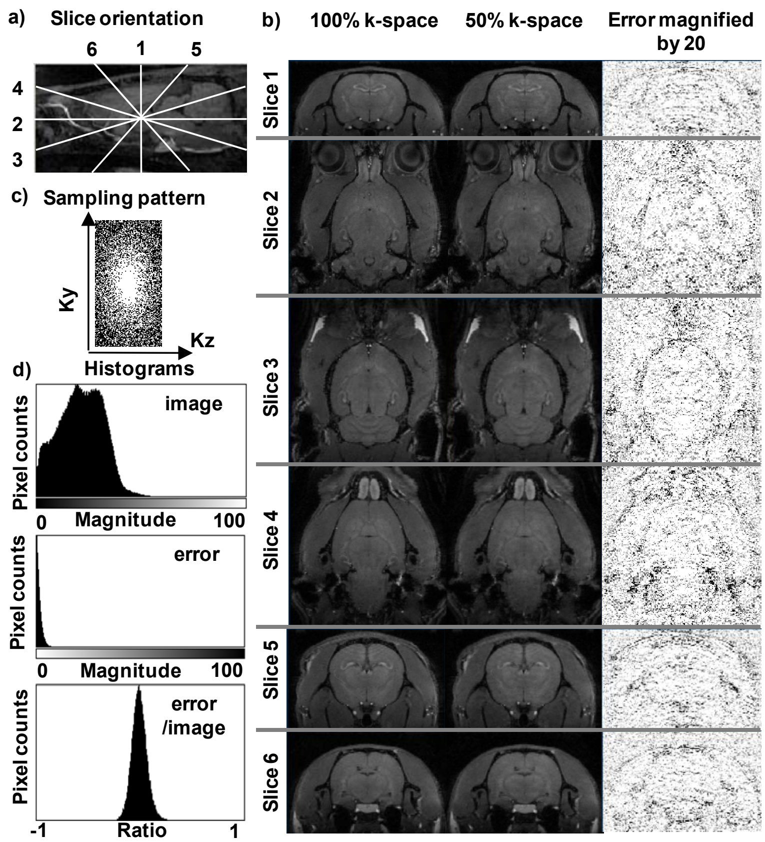


**Fig. 3:** The original 3D image (100% k-space) compare with 50% k-space result. (**a**) The geometry orientations of six oblique views. (**b**) From left to right: 100% k-space images (the original), 50% k-space reconstructed image and error map magnified by 20. (**c**) The 50% unersampling pattern. (**d**) From up to down is histogram of: original image, error and error/image ratio.

**REFERENCES: 1.** Donoho, IEEE Trans. Inform. Theory 52: 1289-1306 (2006); **2.** M. Lustig, Magn Reson Med 58:1182-1195 (2007); **3.** Lee, Magn Reson Med 34:308-12 (1995)