Theoretical basis of projection based shim estimation

D. N. Splitthoff[1], and M. Zaitsev[1]
[1]Dept. of Radiology, Medical Physics, Unversity Medical Center Freiburg, Freiburg, Germany

Introduction: The usage of projections for calculating changes in B0 inhomogeneities has been suggested in several publications. The approach consist in calculating phase differences in image space (e.g. [1,3]), or, equivalently, shifts in k-space (e.g. [2]). However, previous works lack a proper and consistent description of the mathematical equations, relevant for detection of B0 inhomogeneities. Here a more detailed analysis is given, connecting the rather pragmatic calculations previously used with the framework of linear algebra. The results reveal specific requirements for projection based shimming, previously ignored.

Concepts: In general the projection of a 2D magnetisation distribution at time t_n, P_n, can be written as shown in Eq. (1), where $m(x,y)$ refers to the absolute magnetisation density, $\beta(x,y)$ to the sum of transmit and receive B1 phase, $R2^*$ to relaxation and $\omega(x,y)$ to the B0 inhomogeneities to be detected. Performing the complex conjugate multiplication of two projections with different t_n ($P_{21}(x)$, c.f. Eq. 2), in analogy to field map calculations, gives meaningful results only under the following strong assumptions: the relaxation has to be uniform and negligible for $\Delta t = t_2 - t_1$, otherwise the signal content will be different in the two projections; similarly, the inhomogeneities cannot be of arbitrary order or the signal dephases. Using a small angle approximation and ignoring relaxation the phase difference $P_{21}(x)$ can then be expressed as in Eq. 3, where $z(x,y) = -R2^*(x,y) + i\omega(x,y)$. Calculating the angle of Eq. 3 yields approximately Eq. 4. The result leads to a series of important insights:

1. the phase difference corresponds to a normalised weighted integral of the inhomogeneities. When written as a matrix multiplication, it becomes obvious that, due to the weighting, only values relative to a reference can be obtained.
2. the weighting depends on $t_1 z(x,y)$ (Eq. 4), and therefore does not stay constant. This conflicts with the previous point. The same holds for motion.
3. unless for a perfectly homogeneous elliptical magnetisation distribution, aligned with the coordinate axes, the phase seen on one axis can be influenced by inhomogeneities along the other direction. This cross talk can be understood considering Eq. (4) and thinking of an inhomogeneity in Y direction which is weighted and then integrated. Especially the localised sensitivity of the elements of coil arrays are likely to break the assumption of homogeneity.
4. as long as the small angle approximation is justified the phase difference should be approximately linear in time and inhomogeneities.

The consequences are:

1. from insight 1 and 2: avoid using two projections with different t_n along a given axis for the phase difference calculation, as suggested by [1] and [3], but projections with the same t_n over different TRs and detect inhomogeneities relative to some reference measurement; Eq. (4) then changes from Δt to t_1, and $\omega(x,y)$ to $\Delta\omega(x,y)$. Motion correction is paramount in any case, due to the weighting of the inhomogeneities and the fact that the values are relative.
2. from insight 3 and 4: as mentioned the crosstalk can only be neglected under special conditions. If no a priori information is available we suggest to measure the crosstalk and incorporate it into the fitting of the phases, i. e. for fitting an inhomogeneity along Y the corresponding effect on X has to be considered.

$$P_n(x) = \int m(x,y) e^{i\beta(x,y) - t_n R2^*(x,y) + i t_n \omega(x,y)} dy = p_1(x) e^{i\phi(x)} \tag{1}$$

$$P_{21}(x) = \overline{P_1}(x) P_2(x) = p_1(x) e^{-i\phi(x)} \int m(x,y) e^{i\beta(x,y) - t_2 R2^*(x,y) + i t_2 \omega(x,y)} dy \tag{2}$$

$$\begin{aligned} P_{21}(x) &\approx p_1(x) e^{-i\phi(x)} \int m(x,y) e^{i\beta(x,y) + t_1 z(x,y)} (1 + i\Delta t_1 \omega(x,y)) dy \\ &= p_1(x) e^{-i\phi(x)} p_1(x) e^{i\phi(x)} \\ &\quad + i\Delta t \int \omega(x,y) m(x,y) e^{i\beta(x,y) + t_1 z(x,y) - i\phi(x)} dy \end{aligned} \tag{3}$$

$$\begin{aligned} \sphericalangle P_{21}(x) &= atan\left(\frac{imag(P_{21}(x))}{real(P_{21}(x))}\right) \\ &\approx \frac{\Delta t \int \omega(x,y) m_1(x,y) \cos(\beta(x,y) + t_1 \omega(x,y) - \phi(x)) dy}{p_1(x)} \end{aligned} \tag{4}$$

$$P_1^{\pm}(x) = \int m(x,y) e^{i\beta(x,y) + t_1 z(x,y) \pm k_y y} dy \tag{5}$$

$$\begin{aligned} P_1^{+}(x) - P_1^{-}(x) &= \int m(x,y) e^{i\beta(x,y) + t_1 z(x,y)} (i2sin(k_y y)) dy \\ &\approx i2k_y \int y\, m(x,y) e^{i\beta(x,y) + t_1 z(x,y)} dy \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{1}{2k_y} imag(e^{-i\phi(x)} (P_1^{+}(x) - P_1^{-}(x))) \\ &\approx \int y\, m_1(x,y) \cos(\beta(x,y) + t_1 \omega(x,y) - \phi(x)) dy \end{aligned} \tag{7}$$

Methods: In order to fulfill the requirements we used a modified Echo Planar sequence (EPI), where prior to the imaging readout the acquisition of one projection along Y and one along X was inserted. The concept was tested in axial acquisitions of a large bottle phantom on a 3T TIM Trio (Siemens Healthcare, Erlangen, Germany) using a twelve channel head array. The phantom was purposely positioned off-centre (Figure 1) to cause effects in addition to coil sensitivities alone. During the first two TRs the effects of X inhomogeneities on Y was calibrated, during the next two TRs the effect of Y on X. This was done by introducing small gradient moments before the projections on the respective conjugate axis, once with positive and once with negative polarity (Eq. 5). Subtraction leads to estimates of the effects of the linear terms on the respective other axis, c.f. Eq. 6 and 7; k_y was chosen such that a dephasing of 45° was achieved on the edges of the field of view (FOV). The fifth TR was chosen to be the reference for the inhomogeneities; in order to improve stability of the crosstalk estimates the zeroth order fluctuations during the first four TRs were estimated based on the respectively unmodified projections and the reference. From the sixth projection onwards the inhomogeneities were calculated relative to the fifth, using a magnitude weighted fit of the phase with the cross talks taken into account. A series of nine measurements was performed, with 10 TRs each. The shim settings on X and Y were altered from one measurement to the next as shown in Table 1; values from all scans were calculated relative to the reference of the first scan. The navigator resolution was set to 64px, with t_1=2.31ms (Y) and t_2=3.27ms (X). The imaging parameters: TR=1s, 13slices, resolution 64x64, FOV 0.224x0.224m². Processing was performed in Matlab (The MathWorks Inc, Natick, MA).

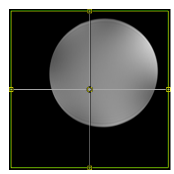

Figure 1: the cylindrical phantom in the transversal slice. The iso-centre is indicated by the crossing of the lines

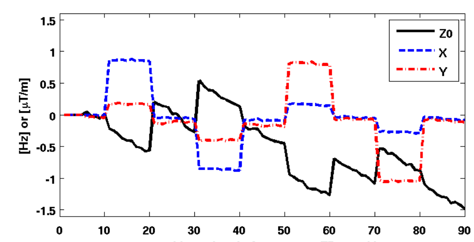

Figure 2: results from fitting ignoring the cross talk; during the gray areas calibrations scans were acquired. The curves do not reflect properly the applied shim changes (Table 1) and show cross talk.

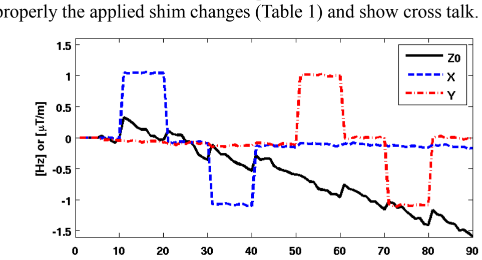

Figure 3: results taking into account the cross talk. The shim changes from Table 1 are much better reflected, along with a linear drift of Z0.

#meas	1	2	3	4	5	6	7	8	9
X (µT/m)	0	+1	0	-1	0	0	0	0	0
Y (µT/m)	0	0	0	0	0	+1	0	-1	0

Table 1: the shim modifications for the nine measurements (10 TRs each), relative to the first measurement.

Results & Discussion: Figure 1 shows the position of the phantom, shifted to the upper right corner. For this configuration a magnitude weighted fit of the phase, ignoring the crosstalk of the axes, is shown in Figure 2; as can be seen all three axes are influenced by the shim changes. Taking into account the cross talk estimated from the calibration TRs of the first measurement leads to results shown in Figure 3. The frequency offset shows a clear drift over time probably caused by shim heating, which is detected independent of the applied shim settings only in Figure 3.

In this work we have presented important considerations which have to be taken into account when using projections for shimming, when no further a priori knowledge is available, along with a possible remedy. The method relies on the assumption that the changes in inhomogeneities, $\Delta\omega(x,y)$, are of low orders, which is an assumption implicitly made as well by previously published approaches [1-3]. Unlike for the FASTMAP [4] approach where the phase is observed along a thin bar, the whole 2D slice is taken into account, which should lead to values that represent better the overall inhomogeneities; this will have to be confirmed in following studies. Another topic of investigation will be in-vivo measurements, where breath holding during the first TRs will probably be required.

References: [1] Ward et al. MRM 48, 771-780, 2002; [2] van der Kouwe et al., MRM 56, 1019-1032, 2006; [3] Splitthoff et al., ISMRM Berlin 2007, #985; [4] Gruetter, MRM 29, 804-811, 1993.

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