Incorporating Support Constraints for Sparse Regularization Reconstruction

F. Lam[1,2], R. Subramanian[3], D. Xu[3], and K. F. King[3]

[1]Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, United States, [2]Beckman Institute, University of Illinois at Urbana-Champaign, Urbana, IL, United States, [3]GE Healthcare, Waukesha, WI, United States

Introduction: Conventional compressed sensing (CS) based reconstruction exploits no extra prior information except the sparsity of the MR image in certain transformation domain to enable high quality reconstruction with sparsely sampled data [1]. However, various other kinds of prior information about the image to be reconstructed can be taken advantage of. We propose a new method to incorporate support information into sparse regularization reconstruction to enable even higher acceleration factor. Similar work can be found in [2-3], where support information is applied in a harsh 0-1 mask, which is not robust to compressible signal and in the presence of noise. In contrast to previous work, our method divides the support into three categories and softens the constraints applied with support information. A mixed L1-L2 regularization is established with support constraints incorporated. This method can also be extended to use support information from various transformation domains. Improved reconstruction quality using the proposed method will be demonstrated at high acceleration factor with a non-contrast MRA data set and a brain imaging data set.

Theory: Given a low resolution estimate of the target image or a high resolution reference image, their support information in certain transformation domain can be divided into three categories: (i) "Sparse/Compressible" support, which corresponds to the most significant coefficients at only a small portion; (ii) The image support (excluding the first part), which is not sparse; (iii) The background, which should be zero ideally but is not due to noise in practice. Fig. 1 gives an example of the three types of the support in a MRA image. The region corresponding to the bright vessels (blue) is a sparse support. The region for the dimmed static tissues (green) is a nonsparse support but still contributes an important energy portion to the data, which means it is not desirable to penalize this region too harshly. The red region is the background. According to this categorization, we propose the following image reconstruction formulation, where $M_{1/2}$ are diagonal matrices with weights for transform coefficients as diagonal entries:

Fig.1: Demonstration of different support types.

$$\rho^* = \arg\min_{\rho} \|F_u\rho - d\|_2^2 + \lambda\|M_1 W\rho\|_1 + \mu\|M_2 W\rho\|_2^2 \quad (1).$$

We apply different weights for different supports (Supp (i) stands for support type (i) mentioned above) as follows: $M_{1/2,jj}$ = $a_{1/2}$, j in Supp(iii); $b_{1/2}$, j in Supp(ii); $\sigma_{1/2}$, j in Supp(i); $a_{1/2} > b_{1/2} \gg \sigma_{1/2}$. In Eqn. (1), F_u is the undersampling Fourier operator and d is the measured data. W is a certain sparsifying transform, in whose domain the support information is detected. λ and μ are regularization parameters. The support detection scheme is a thresholding procedure based on two thresholds. It can be written as: Supp(i) = $\{k, |[W\rho]_k|>\varepsilon_2\}$ and Supp(ii) = $\{k, |[W\rho]_k|>\varepsilon_1\}$ with $\varepsilon_1< \varepsilon_2$. ε_2 is calculated using the "First significant jump" scheme proposed in [2], which is more stable in detecting sparse support for compressible signal and ε_1 is obtained by empirically scaling ε_2 with a factor. Multiplicative half-quadratic approximation is used to solve the optimization problem. The choice of regularization parameter is done by running the reconstruction several times and choosing the value correspond to the image with best quality.
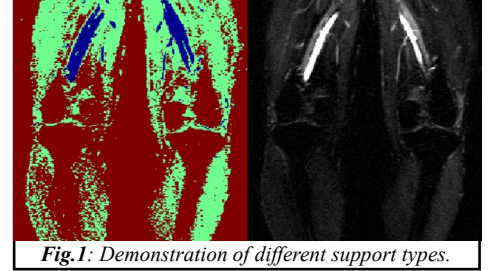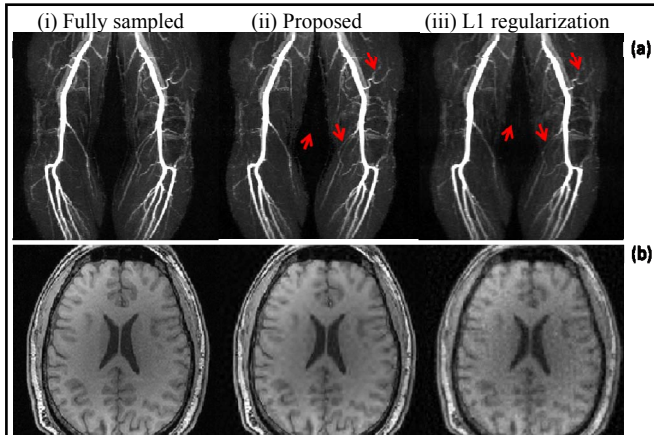

Fig.2: Reconstruction comparisons for two application cases:(a)MIP images for MRA reconstruction; (b)Brain image reconstruction using regularization on wavelet coefficients.

Methods: We have evaluated the proposed method in several scenarios. One of them is a non-contrast leg MRA experiment. Two 3D images at diastole and systole are acquired respectively in this case. Then an MRA image with bright vessels and dimmed static tissues can be reconstructed from subtraction data. The fully sampled data at matrix size 256x256x64 were acquired using a 3D FSE sequence [4]. The subtraction data were then undersampled by a factor of 5 with a random Gaussian sampling pattern in ky-kz plane. Proposed reconstruction was applied with W being an identity matrix and $a_1 = a_2 = 2, b_1 = b_2 = 1, \sigma_1 = \sigma_2 = 0.1, \lambda = 5\times10^{-6}, \mu = 5\times10^{-5}$. The low resolution image reconstructed from dense low frequency k-space data was used to extract the support information. The other evaluation was done using two brain images with one of them as the reference image. The three types of support in the wavelet domain of the reference were detected and the information is used for reconstructing the other image. The matrix size for this data is 256x256 and it is undersampled by a factor of 6 with the same pattern above. In this case W is a discrete wavelet transform and we set $a_1 = 2, a_2 = 3, b_1 = b_2 = 1, \sigma_1 = \sigma_2 = 0.1, \lambda = 5\times10^{-6}, \mu = 5\times10^{-5}$.

Results: Fig.2-(a) shows the comparison for MRA reconstructions between the proposed method and standard L1 regularization. The maximum intensity projection (MIP) images are shown. At this undersampling rate, the proposed method outperforms L1 regularization in providing much clearer vessel visualization and successfully suppressing the background artifact. The reconstruction comparison for brain image shown in Fig. 2-(b) is also in good agreement with the results shown in Fig. 2-(a). The proposed method significantly reduced the artifact result from L1-wavelet reconstruction.

Conclusion: A sparse regularization reconstruction scheme with support constraints is proposed. Support information is divided into three categories and incorporated as a mixed weighted L1-L2 regularization formulation. Better reconstruction is achieved with the same acceleration factor compared with conventional L1-regularization reconstruction. Obtaining better support detection scheme, weighting assignments and refining the choice of regularization parameters will be interesting topics to pursue in the future.

Reference: [1] Lustig M *et al, MRM* 58: 1182-1195, 2007. [2] Y. Wang, *et al, RICE CAAM,* 462-490, 2009. [3] N. Vaswani *et al, IEEE TSP* 58, 4595-4607, 2010. [4] Mitsue Miyazaki *et al, Radiology* 248: 20-43, 2008.