Group Sparse Reconstruction of Vector-Valued Images

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Overview: Sparsity promotion is a reconstruction strategy that has proven very effective for accelerated MRI applications, permitting images to be accurately formed using much less data than traditionally required (e.g., by linear methods) [1]. Of particular success have been techniques that exploit redundancy in the temporal or parametric series dimension [2-8], such as the periodicity of cardiac motion via Fourier transform. However, not every MRI series has an obvious sparsifying transformation along the temporal or parametric dimension. One example is a radiologist's preferred sequence of acquisitions for certain pathology, which can, amongst others, consist of a mixture of various T_1-weighted, T_2-weighted (with and without FLAIR), PD, DWI, and pre- and post-contrast images. Other examples are diffusion tensor (DTI) and spectrum (DSI) imaging. Prior works have demonstrated that temporal or parametric correlations between spatially-similar voxels can be exploited in a blind fashion for improved denoising or reconstruction performance [9-11]. More recently, a generalization of this notion to so-called joint or group sparsity models has been proposed [12,13], where the target signal is encouraged to be sparse in one dimension and correlated (i.e., dense) in another. This idea has so far been applied to the coil-by-coil parallel image reconstruction problem, where it is assumed that coil images share the same spatial support (either intrinsically [14] or in some transform domain [15,16]), as well as cardiac imaging [17], where it is assumed that the image series can be explicitly spatially partitioned into groups such that the number of groups not exhibiting a highly-compact *x-f* space would be sparse. Motivated by [11], we demonstrate that joint sparsity can not only be utilized for highly-undersampled image reconstructions but also that it offers the potential for significantly improved performance over image-by-image reconstructions, which are inevitably necessary when a temporal or parametric sparsifying transformation is not available.

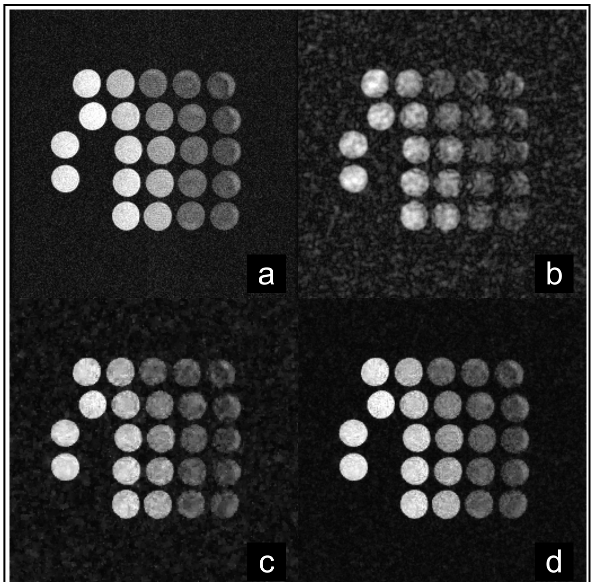

Fig 1. Example reconstruction of ten-angle variable flip angle image series (a) recovered from only 20% of its Fourier samples by zero-filling (b), image-by-image ℓ_1-minimization with finite difference sparsity (c), and the proposed vector ℓ_1-minimization (d).

Methods: Recall that, for MRI, the ideal sparsity-driven reconstruction involves minimization of a least-squares fidelity term regularized by the ℓ_0-norm, which is simply a count of the number of non-zero signal components. For the scalar problem, determination of whether the signal is non-zero at a point simply involves checking its magnitude. To generalize this notion to vector-valued images, some measure of the length of the vector associated with a point must be used. Note that a (possibly transform) signal that is spatially sparse but temporally or parametrically dense implies that that signal possesses strong spatiotemporal correlations. This suggests usage of the Chebyshev or ℓ_∞-norm as the measure of vector density. Thus, the vector generalization of the ℓ_0-minimization problem is:

$$u = \arg\min_u \left\{\alpha\|\Psi u\|_0 + \|Au-g\|_2^2\right\} \quad \rightarrow \quad u = \arg\min_u \left\{\alpha\sum_x 1\left(\|S_x\Psi u\|_\infty > 0\right) + \|Au-g\|_2^2\right\} \tag{1}$$

where u is the target signal, Ψ is the spatial sparsifying transform, α is a regularization parameter, A is the acquisition transform, and the observed signal $g=Af+n$. S_x is a binary selection operator that isolates the vector associated with the spatial position index by x. As the ℓ_0-minimization problem is intractable due to its non-convexity, its closest convex approximation, the ℓ_1-minimization (the ℓ_1-norm is simply the sum of absolute values of a signal) is often used as a surrogate [1]. This convex relaxation can also be applied to the vector generalization of the ℓ_0-minimization problem; however, the embedded ℓ_∞-norm is non-differentiable which limits the range of numerical methods than can be used to solve this problem. Whereas the ℓ_1-norm is chosen as the closest convex and non-smooth approximation of the ℓ_0-norm, the ℓ_∞-norm can be replaced by its farthest convex and smooth approximation, the ℓ_2-norm.

$$u = \arg\min_u \left\{\alpha\|\Psi u\|_1 + \|Au-g\|_2^2\right\} \quad \rightarrow \quad u = \arg\min_u \left\{\alpha\sum_x \|S_x\Psi u\|_2 + \|Au-g\|_2^2\right\} \tag{2}$$

The relaxed vector sparsity problem can be readily solved using descent based methods, noting that, for (2),

$$\nabla_{\bar{u}} J(u) = \frac{\alpha}{2}\Psi^*\left(\sum_x S_x S_x \|S_x\Psi u\|_2^{-1}\right)\Psi u + A^*(Au-g) = 0 \tag{3}$$

Example: Fig. 1 displays a 256x256 variable flip angle SPGR image sequence (GE Signa 2.0T, v14.0, TR/TE=18.0ms/4.01ms, head array, single-channel, FA={2,5,10,15,20,25,30,35,40,45}) reconstructed after retrospective 80% variable density undersampling (a different random instance was used for each image) using both a standard ℓ_1-minimization approach and the proposed vector generalization. For the sake of demonstration, we assume that an efficient means of sparsifying such signals along the parametric dimension is not known (alternatively, see [7,8]). Finite spatial differences were used as the sparsifying transformation. The problem in (3) was solved using a generalization of the quasi-Newton method described in [18], with 10 outer and 20 inner iterations performed. The regularization parameter, α, was manually optimized for each reconstruction. Although the standard ℓ_1-minimization reconstruction (Fig. 1c) demonstrates improved sharpness and support recovery relative to the zero-filled reconstruction (Fig. 1b), the result nonetheless contains a high degree of artifact likely due to the high noise level in the original signal. Conversely, the proposed vector reconstruction exhibits much stronger fidelity to the fully-sampled image (Fig. 1a) with less artifact and improved background-to-foreground contrast.

Discussion: In this work, we have investigated a generalization of the sparsity-driven reconstruction paradigm for vector-valued images and demonstrated its feasibility for the highly-undersampled image reconstruction problem. While any existing temporal or parametric prior information about a target image should obviously be exploited during reconstruction, if no such information is available for an image series then joint sparsity may offer a means for improving reconstruction performance beyond what standard sparsity-driven methods can offer.

References: [1] M. Lustig et al., MRM 58(6):1182-1195; [2] M. Lustig et al., Proc. ISMRM 2006, p. 2420; [3] U. Gamper et al., MRM 59(2):365-373, 2008; [4] M. Doneva et al., Proc. ISMRM 2009, p. 824; [5] H. Jung et al., MRM 61(1):103-116, 2009; [6] R. Otazo et al., Proc. ISMRM 2010, p. 344; [7] J. Velikina et al., Proc. ISMRM 2010, p. 4865; [8] J. Velikina et al., Proc. ISMRM 2010, p. 350; [9] G. Gerig et al., IEEE TMI 11(2):221-232,1992; [10] J. Haldar et al., Proc. ISMRM 2008, p. 141; [11] J. Haldar and Z. Liang, Proc. IEEE ISBI 2008, p. 752-755; [12] M. Yuan and Y. Lin., J. Royal Stat. Soc. B 68(1):49-67, 2006; [13] J. Tropp et al., Proc. IEEE ICASSP 2005, p. 721-724; [14] D. Liang et al., Proc. ISMRM 2009, p.377; [15] R. Otazo and D. Sodickson, Proc. ISMRM 2009, p.378; [16] M. Lustig et al., Proc. ISMRM 2009, p.379; [17] M. Usman et al., Proc. ISMRM 2010, p. 545; [18] J. Trzasko and A. Manduca, IEEE TMI 28(1):106-121, 2009.