GAUSSIAN SCALE MIXTURE-BASED JOINT RECONSTRUCTION OF MULTICOMPONENT MR IMAGES FROM UNDERSAMPLED K-SPACE MEASUREMENTS

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Introduction: Undersampling the k-space measurements can reduce the acquisition time in magnetic resonance imaging (MRI) at the cost of introducing the aliasing artifacts. The sparsity of magnetic resonance (MR) images in wavelet transforms shows promising results to suppress these artifacts [1]. Recently, the Gaussian Scale Mixture (GSM)[2] for modeling dependency of wavelet coefficients of single image improve the wavelet-based reconstruction[3-5]. In this paper, we consider the cases that MR study is comprised by many different types of images of the same patient (e.g. T1, T2, Proton density-PD, etc). By modeling the dependency of wavelet coefficients of the multi-component images, a multicomponent GSM (mGSM)-based iterative algorithm is proposed to jointly reconstruct these MR images from undersampled k-space measurements. Simulations demonstrate that this model can improve the reconstructed MR images than the traditional wavelet-based iterative hard thresholding (IHT) does for each image separately.

Methods: Let the image to be reconstructed be $\mathbf{x}$ whose wavelet coefficients are $\boldsymbol{\alpha} = \boldsymbol{\Psi}^T\mathbf{x}$. The $\boldsymbol{\Psi}$ denotes the wavelet basis and $\boldsymbol{\Psi}^T$ means the forward wavelet transform. The conventional compressed sensing MRI (CS-MRI) reconstruct image by solving $\arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{F}_u\mathbf{x}\|_2^2 + \lambda\|\boldsymbol{\Psi}^T\mathbf{x}\|_1$. This can be interpreted as maximum a posteriori (MAP) estimation from the Bayesian perspective [3,5] as $\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} \log p(\mathbf{x}|\mathbf{y}) = \arg\max_{\mathbf{x}} \left[\log p(\mathbf{y}|\mathbf{x}) + \log p(\mathbf{x})\right]$ where the second term is related the l_1 norm term following $p(\mathbf{x}) \propto \exp\left(-\lambda\|\boldsymbol{\Psi}^T\mathbf{x}\|_1\right) = \exp\left(-\lambda\|\boldsymbol{\alpha}\|_1\right)$. For the multiple component MR image, the wavelet coefficients of all the images at a given spatial position j in the same subband are grouped into a B-dimensional vector $\boldsymbol{\alpha} = \left[\alpha_j^{s,1} \quad \alpha_j^{s,2} \quad \cdots \quad \alpha_j^{s,B}\right]^T$ **.** Equivalent processing is typically applied to all the wavelet subbands hence s is omitted. Assume the aliased artifact is random Gaussian noise, then the wavelet coefficients of aliased multicomponent images can be expressed as $\mathbf{h} = \boldsymbol{\alpha} + \boldsymbol{\varepsilon} = \sqrt{z}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$. The $\boldsymbol{\theta}$ is weighted by a scalar random variable z . Both $\boldsymbol{\theta}$ and $\boldsymbol{\varepsilon}$ are assumed to be zero-mean Gaussians with covariance $\mathbf{C}_{\boldsymbol{\theta}}$ and $\mathbf{C}_{\boldsymbol{\varepsilon}}$. The de-aliased wavelet coefficients can be computed [2,3,6] according to

$$\hat{\boldsymbol{\alpha}} = E(\boldsymbol{\alpha}|\mathbf{h}) = \int_0^\infty E(\boldsymbol{\alpha}|\mathbf{h}, z)\, p(z|\mathbf{h})\, dz$$

where E is the expectation operator and $E(\boldsymbol{\alpha}|\mathbf{h}, z) = z\mathbf{C}_{\boldsymbol{\alpha}}\left(z\mathbf{C}_{\boldsymbol{\alpha}} + \mathbf{C}_{\boldsymbol{\varepsilon}}\right)^{-1}\mathbf{h}$. The computation of $\mathbf{C}_{\boldsymbol{\alpha}}$, $\mathbf{C}_{\mathbf{h}}$, $p(z|\mathbf{h})$ and $E(\boldsymbol{\alpha}|\mathbf{h})$ is explained in [3,6]. With a decreasing variance of $\boldsymbol{\varepsilon}$, the reconstructed image is obtained by thresholding on the aliased image $\mathbf{x}$ using the Bayesian Least Squares (BLS)-GSM method[6]. The framework of the proposed method is shown in Fig.1.

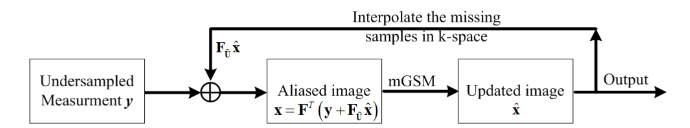

Fig. 1 Reconstruct the multicomponent MR image using mGSM. $\mathbf{F}_{\bar{U}}$ denotes filling the missing measurements in k-space of image $\mathbf{x}$ with the corresponding measurements in k-space of the updated image $\hat{\mathbf{x}}$.

Results: The multicomponent MR images [6,8] utilized in simulation are T1, T2, and PD (low and high flip angle) weighted MR images as shown in the first row of Figure 2. These images are 1mm^3 voxel resolution and 8 bit quantization from the BrainWeb phantom [7]. Each image is radially sampled in k-space with rate 15% and the locations of radial lines for each image are different. The iterative hard thresholding(IHT) for CS-MRI[9] with decreasing factor μ=0.95 of threshold is employed for each image separately to compare the performance with mGSM reconstruction. The initial variance of the aliased image is 10 and the decreasing factor is 0.9. Symlets 4 wavelet with 3 decomposition levels in undecimated form is chosen in both methods. The peak signal-to-noise ratio (PSNR) is adopted to quantify the difference between the MR images reconstructed from fully sampled and undersampled k-space. PSNR is defined as $PSNR = 20\log_{10}\left(255/\sqrt{MSE}\right)$ where MSE is the mean square error between the ground truth and reconstructed multicomponent images $\tilde{\mathbf{x}}$ and $\hat{\mathbf{x}}$. Simulations show that the proposed method produces high fidelity images and better PSNR.

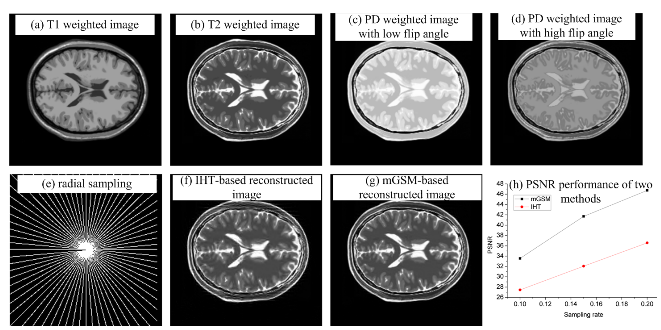

Fig.2 The reconstructed images with mGSM-based and iterative hard thresholding-based methods. (f) and (g) are reconstructed when sampling rate is 0.15.

Conclusions and Discussion: The mGSM is introduced to model the dependency of wavelet coefficients of multicomponent MR images and a iterative method is proposed to joint reconstruct these images from undersampled k-space measurements. Simulations demonstrate the mGSM can improve the reconstruction than the iterative hard thresholding to reconstruct each image separately. For the future work, comparison on reconstruction of vivo data and for noisy measurements will be conducted.

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