MR Compressed Sensing Using FREBAS Transform

S. Ito[1], K. Miyabayashi[2], and Y. Yamada[2]

[1]Research Division of Intelligence and Information Sciencs, Utsunomiya University, Utsunomiya, Tochigi, Japan, [2]Utsunomiya University, Utsunomiya, Tochigi, Japan

Introduction: Compressed sensing (CS) [1] aims to reconstruct signals and images from significantly fewer measurements than were traditionally thought necessary. MRI is a medical imaging tool burdened by an inherently slow data acquisition process. The application of CS to MRI has the potential for significant scan time reductions, with benefits for patients and health care economics. In this paper we present a new CS method based on the FREBAS transform which we have proposed as a new kind of multi-resolution image analysis. The algorithm and the performances of proposed method were demonstrated and it was shown that proposed CS method can achieve a reduction factor higher than the standard CS method.

Theory: According to the CS theory, a signal x with a sparse representation in the basis Ψ, can be recovered from the compressed measurements $y=\Phi x$, where Φ is K x N matrix (k<<N), if the Φ and Ψ are incoherent. The image is reconstructed from the undersampled k-space data by solving the nonlinear optimization problem: minimize $\|\Psi x\|_1$ subject to $\|\Phi x-y\|_2 < \varepsilon$, where ε is a small constant.

In our work, we adopt FREBAS transform[2] as a sparsifying transform matrix. FREBAS transform is made up of two different algorithm of Fresnel transform which allows optional scaling of images. Figure 1 shows an example of FREBAS transformed signal. FREBAS transform is different with wavelet transform in the sense (1) FREBAS transform with scaling parameter D decompose the input image into D^2-number of images having the same scale size of $1/D$, (2) calculation consists of few times of FFTs and quadratic phase modulations only, convolution integral is not included in the calculation, (3) scaling parameter D can take the value not only the integer but also real number. Minimizing $\|\Psi x\|_1$, we use iterative soft thresholding method[3] for faster calculation. The iterative procedure is written as follows;

$$x_{i+1/2} = \psi^{-1} \mathrm{T_h} \psi x_i \qquad (1)$$

$$x_{i+1} = \mathrm{E}^{-1}\left[\mathrm{E} x_{i+1/2}(1-\mathrm{M}) + \mathrm{M}x\right] \qquad (2)$$

where, Ψ is the FREBAS transform, T_h is a thresholding function, E is NxN size Fourier encode matrix and M is the sampling trajectory matrix. The starting condition of Eq.(1) we used zero-filled reconstructed image for x_0. Eq.(2) shows the operation that the signal on the sampling trajectory is replaced by the collected signal in each iterations. **Figure 2** shows the iterative procedure of our method. The image (b) is FREBAS transformed and then small amplitude signals under the threshold level T_h are shrinked. Inversely FREBAS transformed image (e) was Fourier transformed and then the signal on the sampling trajectory was replaced by the acquired signal (a). The threshold value of function T_h on the FREBAS transformed space decreases as the number of iteration increases.

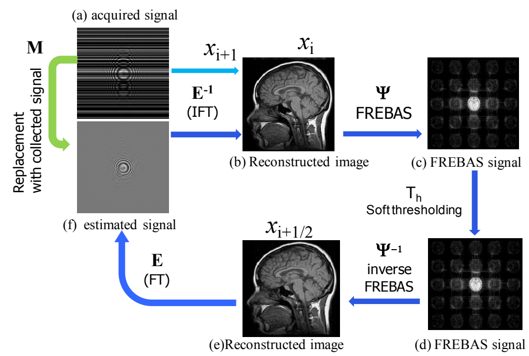

Fig. 1 Schematic of FREBAS Compressed Sensing

Results and Discussion: MR image data were collected on Toshiba 1.5T MRI scanner with 3D gradient echo sequence(TE/TR=40/50ms, 256x256 matrix size, slice thickness 1.5mm x 50slices). We used Cartesian sampling because it is by far the most widely used in practice. k-space signal for the phase encoding direction except for the central region is randomly picked to simulate a given reduction factor. Figure 3 show the RMS of reconstruction error compared with standard CS method proposed by Lustig et al. Figure 4 shows the reconstructed images using the 30% of fully-encoded data. Fig.(a) shows fully-encoded image, and (b),(c) show the images obtained by proposed method after 60-times iterations and CS method by Lustig et al.,respectively. It was shown from Table 1 that the RMS error in proposed method is smaller than that of standard CS. The obtained image (b) has much more details of the object whereas the image (c) is excellent in the sense of smoothness on the image. The reason is in the difference of the manner of multi-resolution image analysis, i.e. FREBAS transform decompose images into DxD number of edge-separated images with DxD number of basis, on the other hand wavelet decomposes images only three components; i.e. High-High, Low-High and High-Low signals. A large number of basis in FREBAS transform can encode complicated structure of images in well sparsified representation and contribute to preserve details of the object.

Conclusion: A new CS technique that uses FREBAS transform as sparsifying transform matrix is proposed and demonstrated. Better reconstruction is achieved with the same acceleration factor compared with conventional compressed sensing.

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References

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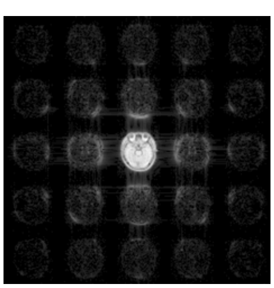


Fig.2 FREBAS transformed signal (scaling coefficient D=5)

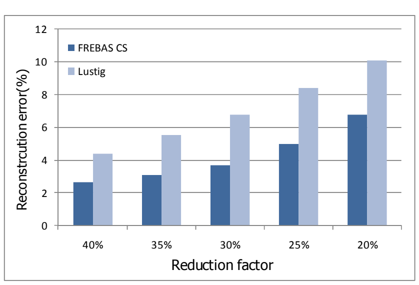
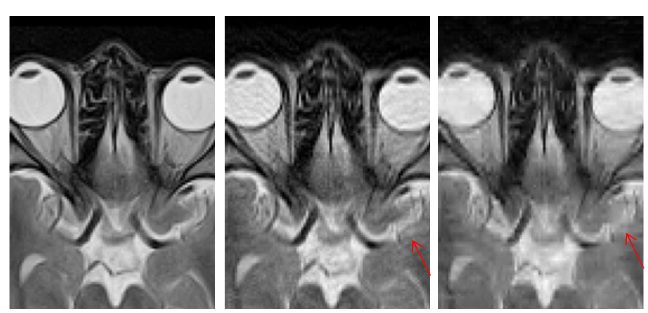


Fig. 3 Comparison of reconstructed error with standard CS.



(a) (b) (c)

Fig. 4 Comparison of reconstructed images using 30% of full data; (a) fully encoded image, (b) FREBAS CS (proposed), (c) standard CS.