Moving Through k-Space by Point Reflections – the TRASE Method

J. C. Sharp[1], and S. B. King[2]
[1]Institute for Biodiagnostics (West), National Research Council of Canada, Calgary, AB, Canada, [2]Institute for Biodiagnostics, National Research Council of Canada, Winnipeg, MB, Canada

Introduction: Standard MRI methods achieve a scanning of k-space by 2 methods: B0-gradients (providing continuous motion) and refocusing (point reflection about the k-space origin). Here we analyze a third option: point reflection about other k-space locations. This is achieved by refocusing with B1-phase-gradients and is the basis of the TRASE (Transmit Array Spatial Encoding) imaging method (1). Our aim here is to formalize the rules governing the construction of k-space trajectories for this type of sequence. These novel k-space operations offer the pulse sequence designer an expanded toolkit for design of sequences exploiting the strengths of both B0-gradient and B1-phase gradient encoding.

Axioms:

[A1] A spin system within a finite spatial volume V can be described by the NMR Bloch equations**.**

[A2] The excitation state of the NMR spin state within V can be described by a k-space distribution $f(\boldsymbol{k})$.

[A3] Over the finite spatial volume V a set of RF field patterns B_1^c can be generated. Each B_1^c ('B1 phase gradient') has uniform magnitude and linear phase gradient as follows: $B_1^c = |B_1^c|e^{i\theta_1^c(r)}$ where $\theta_1^c(\mathbf{r}) = 2\pi\boldsymbol{k}_1^c.\mathbf{r} = \mathbf{G}_1^c.\mathbf{r}$.

Definition: $\boldsymbol{k}_1^c$ is the 'k-space focus' for the phase gradient coil, *c*. (It is a point in k-space)

Definition*: 'Pair sequence*' – a pulse sequence consisting of a finite number of pairs of refocusing pulses.

Some Theorems

Below are statements of some theorems (proofs not given here):

[T1] Refocusing (Point Reflection) Theorem: A 180° refocusing pulse using coil *c*, for excited spins in volume V, results in a transformation of the spins k-space excitation pattern: $f(\boldsymbol{k}_+) = f(-\boldsymbol{k}_- + 2\boldsymbol{k}_1^c)$. i.e. for a point in k-space: $\boldsymbol{k}_+ = -\boldsymbol{k}_- + 2\boldsymbol{k}_1^c$, or $(\boldsymbol{k}_+ - \boldsymbol{k}_1^c) = -(\boldsymbol{k}_- - \boldsymbol{k}_1^c)$ This is a *point reflection*. Note: the final k-space location $\boldsymbol{k}_+$ depends upon both the initial k-space coordinate $\boldsymbol{k}_-$ and on the k-space focus $\boldsymbol{k}_1^c$. In two dimensions, a point reflection is the same as a 180° rotation. In three dimensions, a point reflection can be described as a 180° rotation composed with reflection across a plane perpendicular to the axis of rotation.

[T2] Inverse Theorem: A point reflection is an *involution* (it is its own inverse). The state after a pair of pulses AA is identical to the state before. (Note: A pair AA is not necessarily without purpose, however as data can be collected after the 1st A.)

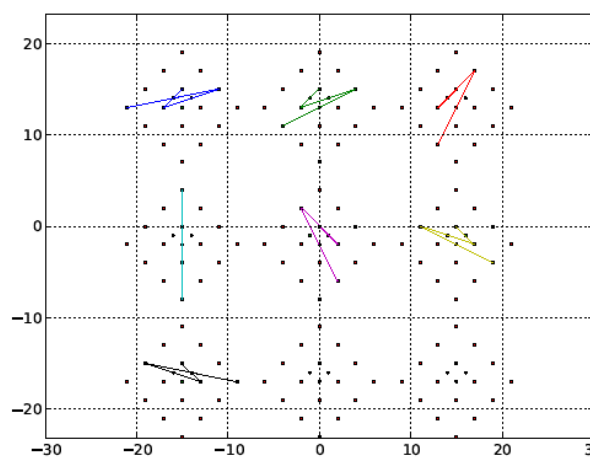
[T3] Pair Refocusing (Translation) Theorem: Two refocusing pulses (point reflections) AB result in a translation of the k-space excitation pattern $f = f(2(k_1^B - k_1^A))$. (*Note*: unlike for [T1] the result is independent of the initial k-space state.)

[T4] Reverse Translation: Reversing a pair (AB → BA) reversed the pair translation vector.

[T5] Sum of Translations: The final destination of a pair sequence is the sum of the pair vectors, without regard to order. (In a pair sequence the order of the pairs affects the route but not the final destination.)

[T6] Two Reflection Points (k-space foci): Given a single excitation location, 2 k-space foci are sufficient to traverse 2 particular lines in k-space.

[T7] Three Reflection Points (k-space foci): Three reflection points are sufficient for the generation of a trajectory in a k-space plane.



The 8 most rapidly outward moving 3-pulse sequences, given 4 k-space foci (coordinates (1,0), (-1,0), (0,1), (0-1), and given excitation to (0,1).

Conclusions: The purpose of this approach is to formalize as far as possible the description of the capabilities and properties of sequences involving k-space point reflections to aid in coil and sequence design. Applications include imaging without B0-gradients and additional encoding mechanism in conjunction with switched-B0-gradients.

References: Sharp JC & King SB Magn. Reson. Med. 2010 63(1) p.151-161.