## Joint Inhomogeneitiy Estimation for Water-fat Separation with Multi-peak Fat Modeling

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**Introduction**: Key to the success of phase-sensitive water-fat separation lies in robust estimation of field inhomogeneities, which remains difficult in many clinically important imaging scenarios. The difficulty often arises when the spectral field-of-view is not sufficient to accommodate field inhomogeneities, causing spectral aliasing. Extensive research efforts have been directed to robust field map estimation by imposing spatial field map smoothness [1-3] and/or deriving a priori likelihood of water and fat existence [4]. This work describes a novel field map estimation technique that systematically incorporates field map smoothness and *a priori* likelihood of field map values via belief propagation (BP) algorithm, which perform joint estimation of field inhomogeneities across 2D image grid (Figure 1).

**Theory**: Feasible field map values at one pixel can be found by minimizing the associated least-squares cost function, which is periodic with the fundamental period equal to the spectral field-of-view $1/\Delta TE$. Taking into account all replicas of feasible field map values inside the worst possible field inhomogeneities range, the estimation problem is to select the correct field map value for each pixel such that the resultant field map consistently separates water and fat across the whole image. In the presence of severe spectral aliasing, it becomes increasingly difficult to produce correct field maps solely based on the spatial smoothness constraint.

In this work, we derive *a priori* likelihood of feasible field map values based on multi-peak fat modeling. Given a series of echo measurements (e.g., 6 echoes) at one voxel, we fit the echoes to the water-only model and multi-peak fat model, and denote the fitting errors of both models as $R_W$ and $R_F$, respectively [4]. *A priori* likelihood $P_\psi$ can then be computed for a feasible field map value $\psi$, based on its corresponding water-fat separation results $W$ and $F$:

$$P_\psi = \begin{cases} \exp\left((R_F - R_W)/\max(R_W, R_F)\right), & \text{if } W > F \\ \exp\left((R_W - R_F)/\max(R_W, R_F)\right), & \text{otherwise} \end{cases}$$

For two neighboring pixels $p$ and $q$, the interaction potential $V_{pq}$ between their feasible field map valus $\psi_p$ and $\psi_q$ is

$$V_{pq} = P_{\psi_p} P_{\psi_q} \exp(-(\psi_p - \psi_q)^2),$$

which captures both the differences between two field map values and their *a priori* likelihood.

The interaction potential can be embedded in the sum-product belief propagation (BP) algorithm which jointly estimates the most likely field map values by passing soft-decision messages among neighboring pixels. Specifically, the message passed from $p$ to $q$ at the $t^{th}$ iteration is given by:

$$m_{p\to q}^t(\psi_q) = \sum_{\psi_p} \left( V_{pq} \prod_{s\in\aleph(p)\backslash q} m_{s\to p}^{t-1}(\psi_p) \right),$$

where $\aleph(p)\backslash q$ denotes the set of neighboring pixels to $p$ other than $q$. After T iterations, a belief is computed for each feasible field map value at all pixels:

$$b_{\psi_q} = \prod_{p\in\aleph(q)} m_{p\to q}^T(\psi_q).$$

Subsequently, for each pixel, the field map value with the maximum belief is selected.

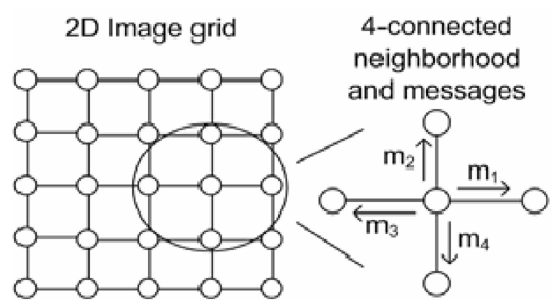


Fig.1 : Illustration of BP message passing on a 4-connected 2D image grid.

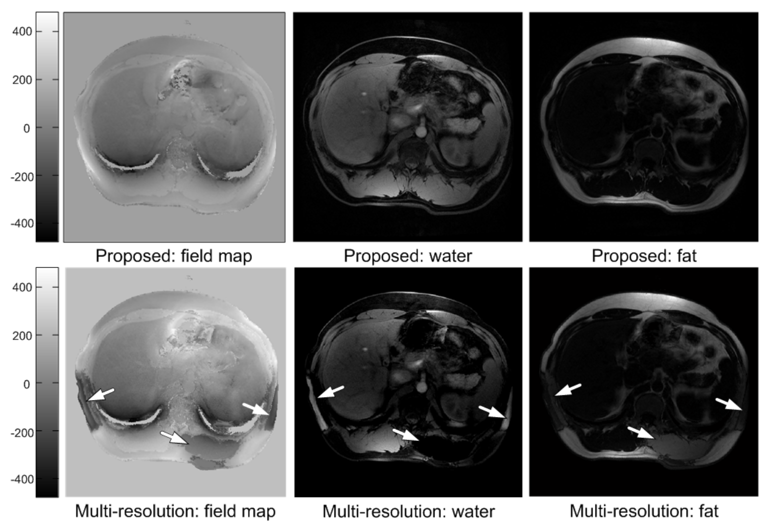


Figure 2: Comparison of field map and water-fat separation results of an abdominal study obtained from the proposed method and multi-resolution field map estimation method.

**Method and Results**: A multi-echo GRE sequence which acquires 6 echoes per repetition was implemented on a Siemens Tim Trio 3T scanner. A quadrature torso coil was used to perform an abdominal study. The relatively long 3.2 ms echo-spacing in this study leads to a spectral FOV of 312.5 Hz, causing spectral aliasing at 3T. Figure 2 show the comparison results of the abdominal study obtained from the proposed technique and the multi-resolution field map estimation method [2] that accounts for the periodicity of feasible field map values. The multi-resolution estimation method fails to keep track of some rapid field map changes and results in the swap of water-fat separation. In contrast, the proposed estimation technique correctly resolves the field inhomogeneities in this challenging case and uniformly separates water and fat.

**Conclusion**: This work demonstrates an effective means to integrate the smoothness constraint with a priori likelihood of field map values. The in-vivo results demonstrate that the proposed new technique outperforms existing techniques in resolving field map ambiguity in challenging scenarios.

**References**: [1] Yu et al. MRM 2005;54:1032-39 [2] Lu et al. MRM 2008;60:236-44 [3] Hernando et al. MRM 2008;59:571-90 [4] Yu et al. ISMRM 2010; pg. 771.