On the impact of regularization and kernel type on SHARP-corrected GRE phase images

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INTRODUCTION – Gradient-echo (GRE) magnetic resonance phase data are proportional to the magnetic field, providing useful information for several applications such as quantitative magnetic susceptibility mapping[1] (QSM), or anatomical contrast analyses[2]. Interpretation of phase information requires phase unwrapping and suppression of the background phase signal that results, e.g., from air- and bone-tissue interfaces. For this purpose, two novel approaches have recently been proposed: Projection onto Dipole Field[3,4] (PDF) and SHARP[5]. PDF fits susceptibility values outside of a volume of interest (VOI) to reproduce the background field within the VOI. Due to this direct and traceable approach PDF corrected phase may be regarded as the gold standard for background contributions that are induced solely by magnetic susceptibilities. The SHARP approach is mathematically more complex but, on the other hand, computationally much more efficient than PDF. The SHARP method is a three-step procedure of convolution with a radial kernel, masking, and deconvolution. The deconvolution step is usually regularized, e.g., using truncated singular value decomposition (TSVD) to account for imperfect phase measurements and non-harmonic B_1-phase contributions. A major pitfall of SHARP is that no reliable phase values can be obtained at the edges of the corrected phase distribution under investigation (e.g. boundaries of the brain) due to aliasing artifacts in these regions. The width of this corrupted region depends on the spatial extend of the applied convolution kernel and, thus,motivates to choose the kernel as small as possible. In this contribution, we present the smallest possible kernel and investigate the impact of both the kernel size and the regularization parameter of SHARP on the resulting background-corrected phase pattern using a numerical brain model.

$$\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\1&-6&1\\0&1&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}$$

FIGURE 2. The three adjacent slices of the simplest discrete Laplacian, which was used in this study.

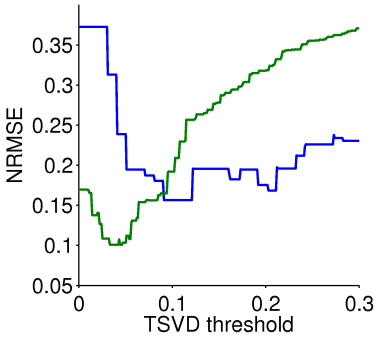

FIGURE 3. NRMSE of the SHARP method with respect to PDF plotted against the regularization parameter for the Laplacian kernel (blue) and a sphere with R=8vx (green).

MATERIALS & METHODS

Numerical Model: T_1 weighted volunteer brain data (1x1x1mm^2) were segmented, and magnetic susceptibilities were assigned to each region. The susceptibility map was then immersed into a homogenous susceptibility distribution similar to the mean susceptibility of the brain model (Fig. 1a). Relatively high susceptibilities were assigned to two small cuboid-shaped regions left and right of the brain to mimic background field contributions (not shown). The field perturbation of the model was computed by fast forward field computation[6] (Fig. 1b).

Processing: In order to generate a gold standard for background field correction PDF was applied to the simulated field perturbation (Fig. 1c). SHARP was successively applied to the simulated field perturbation using smoothly rendered numerical spheres with radii R ranging from 2 voxels (vx) to 10vx (an example with R=8 is shown in Fig. 1e) and regularization parameters ranging from 0 to 3. Furthermore, the discrete Laplacian operator (Fig. 2) was used as a kernel. The Laplacian operator directly solves the differential equation related to the SHARP method[5] and, thus, represents the smallest possible kernel which is equivalent to R=1. For all SHARP-corrected fields the quality was assessed by calculating the normalized root mean square error (NRMSE) with respect to the PDF-corrected image for the same region inside of the brain (indicated by red lines in Fig. 1c-e): $\mathrm{NRMSE} = \|\Delta B_{\mathrm{SHARP}} - \Delta B_{\mathrm{PDF}}\|_2 / \|\Delta B_{\mathrm{PDF}}\|_2$. (1)

kernel	NRMSE	thresh.*100
R=1	0.17	5…25
R=2	0.11	0.2…0.4
R=3	0.13	0.2…0.8
R=4	0.11	0.5…1.5
R=5	0.10	1…2
R=6	0.09	1.5…3
R=7	0.09	2.5…3.5
R=8	0.10	3.5…4.5
R=9	0.11	3.5…6
R=10	0.12	4.5…6.5

TABLE1. Minimal NRMSE and corresponding regularization parameters (TSVD thresholds) obtained with SHARP.

RESULTS – Figure 1 shows the numerical brain susceptibility model (Fig. 1a), the field perturbation with background fields (Fig. 1b), the PDF-corrected field (Fig. 1c), and the SHARP-corrected fields with R=1 (Laplacian; Fig. 1d) and R=8 (Fig. 1e). Minimal NRMSE values and corresponding regularization parameters are shown in Table 1. The deviation between PDF and SHARP was low for all kernels (NRMSE<0.2) when the regularization parameter was chosen appropriately. Best results were achieved for spheres with radii between 5vx and 8vx. The dependence of NRMSE on the regularization parameter was similar for all spherical kernels. The relation between both values is exemplarily illustrated in Fig. 3 for R=8 (green line). The Laplacian was fairly independent from the chosen regularization parameter (>5), which is also illustrated in Fig. 3 (blue line), but resulted in slightly worse quality compared to the spheres (see Table 1). The difference patterns used for the NRMSE calculation (nominator in Eq. 1) with optimal regularization parameters are shown in Fig. 1f and 1g for the Laplacian and R=8, respectively. No high frequency anatomical structures are discernable in these images.

DISCUSSION & CONCLUSIONS – It has been shown that the error due to regularized inversion in the SHARP method is small and that by choosing the simplest discrete Laplacian the size of the convolution kernel can be minimized. Thus, the width of the margin with unreliable data may be reduced to 1-2 voxels, resolving one of the major pitfalls of SHARP compared to PDF. Although the minimal NRMSE of 0.09 (see Table 1) suggests that SHARP results in slightly worse quality compared to PDF, it is unclear whether this has any impact on conclusions drawn from the corrected phase data, e.g., with quantitative susceptibility mapping (QSM). It is, furthermore, unclear whether the (gold standard) PDF-corrected image is really free of artifacts; especially regarding the fact that additional high-pass filtering of the PDF-corrected phase seems to be required in order to suppress non-susceptibility contributions in practice[7]. Future studies will focus on investigating the impact of the observed deviations between SHARP and PDF on QSM-maps.

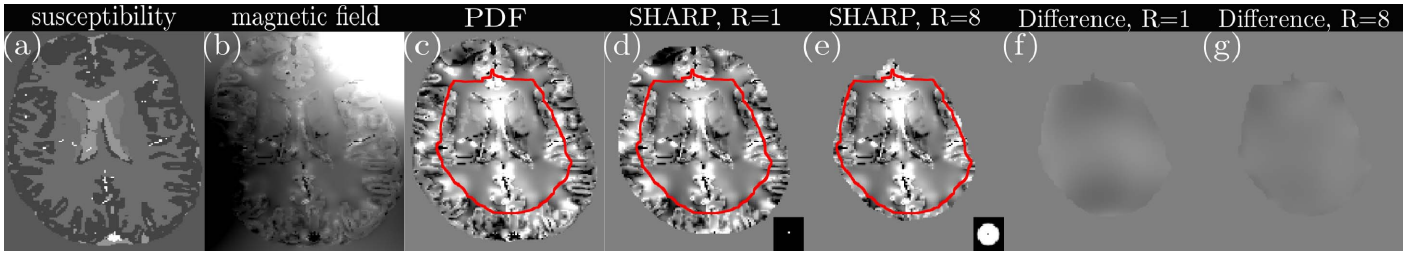

FIGURE 1. (a) Susceptibility distribution (external sources not shown). (b) Magnetic field perturbation due to the susceptibility distribution of the brain and external sources. (c) PDF-corrected phase. (d) SHARP-corrected phase using discrete Laplacian kernel. (e) SHARP-corrected phase using sphere with radius R=8vx. (f) Difference between PDF- and SHARP-corrected phase using discrete Laplacian kernel. (g) Difference between PDF- and SHARP-corrected phase using sphere with radius R=8vx. The Laplacian kernel and the sphere are depicted in the bottom right corners of (d) and (e), respectively.

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