# Harmonic phase subtraction methods are prone to $B_1$ background components

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**INTRODUCTION –** Gradient-echo (GRE) magnetic resonance phase data are proportional to the magnetic field, providing useful information for several applications such as quantitative susceptibility mapping [1], or anatomical contrast analyses [2]. Interpretation of phase information requires phase unwrapping and suppression of the background phase signal that results, e.g., from air- and bone-tissue interfaces. Two novel approaches to estimate these background field contributions have recently been proposed: SHARP [3] and Projection onto Dipole Fields (PDF) [4,5]. SHARP computes the background field using a three-step procedure of convolution, masking, and deconvolution. PDF, on the other hand, fits susceptibility values outside of a volume of interest (VOI) to reproduce the background field within the VOI. In this contribution we demonstrate mathematical equivalence of both methods and investigate the effect of the $B_1$ radio frequency (RF) signal induced phase offset on PDF and SHARP background field estimation.

**THEORY –** SHARP and PDF rely on the same fundamental assumption that the measured phase $\varphi_{\text{total}}$ is a superposition of internal phase contributions $\varphi_{\text{int}}$ and background phase contributions $\varphi_{\text{bkg}}$. The internal contributions are defined to be field perturbations due to heterogeneous susceptibility inside the volume of interest (VOI), e.g., the intrinsic magnetic susceptibility distribution of the brain tissue in the case of a brain examination. The background contributions, on the other hand, are induced by the distribution of dipole sources in the space outside of the VOI, e.g., air-tissue and tissue-bone interfaces in the case of a brain examination. Li and Leigh [6] showed that $\varphi_{\text{bkg}}$ satisfies Laplace's equation (i.e., they are harmonic functions) $\nabla^2\varphi_{bkg}=0$ within the VOI. Thus, both methods, PDF and SHARP, involve solving the following differential equation for $\varphi_{\text{int}}$:

$$\nabla^2\varphi_{total}=\nabla^2\varphi_{\text{int}} \qquad (1)$$

Both PDF and SHARP eliminate only contributions that are harmonic throughout the VOI, and they are therefore theoretically equivalent. However, not all contributions to the background field are necessarily harmonic, such as the phase offset $\varphi_0$ induced by the RF transmit and receive coils (RF offset). $\varphi_0$ can be calculated using the following linear regression on double-echo GRE data:

$$\varphi_0=\varphi_1-\left(\frac{\varphi_2-\varphi_1}{TE_2-TE_1}\right)TE_1,$$

where $\varphi_1$ and $\varphi_2$ are the phase values at echo times $TE_1$ and $TE_2$, respectively: $\varphi_1=\varphi_{\text{total}}(TE_1)$ and $\varphi_2=\varphi_{\text{total}}(TE_2)$.

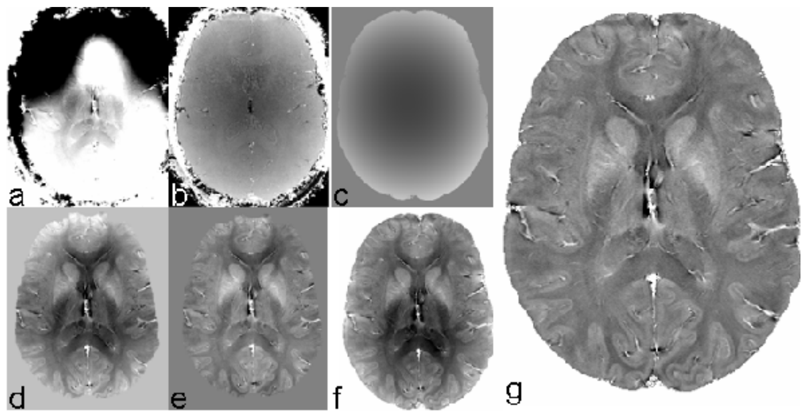


**FIGURE 1**. (a) Unwrapped phase data. (b) Total RF offset. (c) Polynomial fit of (b). (d) SHARP-corrected phase data of (a) without regularization. (e) SHARP-corrected phase of (a) using regularization. (f) PDF-corrected phase of (a). (g) PDF-corrected phase of (a) after subtraction of the RF offset.

**MATERIALS & METHODS**

*Data Acquisition and Pre-Processing:* High-resolution volunteer GRE data of the whole brain were acquired with the ToF-SWI sequence [7] ($TE_1/TE_2$=3.4ms/20ms, TR/FA/$BW_2$=30ms/15°/80Hz/px, voxel size 0.6×0.6×0.6 mm³, 75% PF in phase and slice encoding direction) on a 3T MR-scanner (Tim Trio, Siemens Medical Solutions) using a 12-channel head-matrix coil. Multi-channel phase images were combined using uniform sensitivity reconstruction [8] and 3D phase unwrapping [9] was applied. The RF offset, $\varphi_0$, was calculated according to the above-mentioned equation and $4^{\text{th}}$-order polynomial fitting was applied on a slice-by-slice basis to remove noise.

*Numerical Model:* As a reference model, $T_1$-weighted volunteer brain data (1×1×1 $\text{mm}^3$) was segmented; magnetic susceptibilities were assigned, and the resulting susceptibility map was immersed in a susceptibility similar to the mean susceptibility of the brain. Background field contributions were mimicked in a second model by assigning high susceptibilities to two small cuboid-shaped regions left and right to the brain. The field perturbations for both models were computed by fast forward-field calculation [10].

*Processing*: PDF correction was performed on the original unwrapped *in vivo* data (Fig. 1a), RF offset-subtracted *in vivo* data (not shown), and model data (Fig. 2a). In the *in vivo* processing, the extracted VOI only included brain tissue regions, while for the numerical model the VOI was expanded to include also sources at the surface of the brain, ensuring that the corrected test model phase data were comparable to the reference model data. The SHARP method was applied (spherical shell; radius/thickn.=5/1 vx.) with and without TSVD regularization (threshold=0.01) to both the *in vivo* phase data (Fig. 1a) and the model data (Fig. 2a).

**RESULTS –** Figure 1 depicts representative slices of the *in vivo* data. The RF offset (Fig. 1b,c) shows a dark minimum in the center. This is also discernable in both the SHARP-corrected phase without regularization (Fig. 1d) and the PDF-corrected phase image without subtraction of the RF offset (Fig. 1f). This minimum is removed if the RF-offset is corrected prior to PDF-correction (Fig. 1g) or SHARP is applied with regularization (Fig. 1e). Figure 2 illustrates the model phase with background fields (Fig. 2a), the PDF-corrected model phase (Fig. 2b), and the deviation (Fig. 2c) of this correction from the reference phase (not shown), which is fairly flat and does not show any fine scale artifacts. The difference between images generated with PDF and SHARP, respectively, depicts minor low frequency inhomogeneities (not shown).

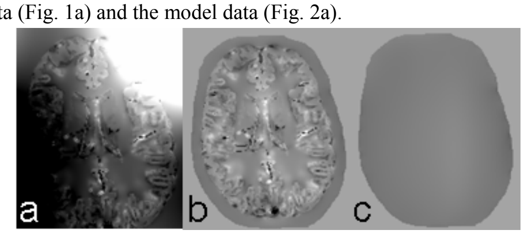


**FIGURE 2.** (a) Model phase data. (b) PDF-corrected model data. (c) Subtraction image of (b) from reference data (scale: [-2.92 1.08] Hz).

**DISCUSSION & CONCLUSIONS –** PDF and SHARP were demonstrated to be mathematically equivalent. Minor (low frequency) differences between the images generated with SHARP or PDF are supposed to result from the different approaches for solving Eq. (1). The fact that the dark minimum remaining in the PDF-corrected phase (Fig. 1f) is eliminated when the RF offset is subtracted from the phase prior to PDF (Fig. 1g) indicates that this contribution is induced by the non-harmonic components of the RF offset. The SHARP method was able to compensate for the RF offset when regularization was applied (Fig. 1e). It is therefore evident that harmonic component subtraction methods (without regularization) cannot eliminate the RF offset which is especially prominent at ultra-high field strength. The numerical simulations indicated that both PDF (Fig. 2c) and SHARP (not shown) are able to suppress contributions from sources outside the VOI without introduction of fine-scale artifacts. Hence, the RF offset has either to be eliminated prior to these methods or has to be accounted for, e.g., with regularization strategies.

**REFERENCES –** [1] Schweser F, et al. *Med Phys*, 37(9):5165-5178, 2010. [2] Deistung A, et al. *Magn Reson Med*, 60(5):1155-68, 2008. [3] Schweser F, et al. *NeuroImage* (in press). [4] Liu T, et al. *Proc ISMRM,* p141, 2010. [5] de Rochefort, et al. *Magn Reson Med*, 63(1):194-206, 2010. [6] Li L and Leigh J. *J Magn Reson*, 148(2):442-8, 2001. [7] Deistung A, et al. *J Magn Reson Imaging*. 2009;29(6):1478-84. [8] Ros C, et al. *ECIFMBE 2008, IFMBE Proceedings 22*. 2009:803-6. [9] Abdul-Rahman HS, et al. *Appl Optic*, 46(26):6623–6635, 2007. [10] Marques JP and Bowtell R. *Concepts Magn Reson B Magn Reson Eng*, 25B(1):65–78, 2005.