Iterative Compressed Sensing Reconstruction for 3D Non-Cartesian Trajectories without Gridding & Regridding at Every Iteration

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INTRODUCTION: Undersampling with non-Cartesian k-space trajectories causes incoherent artifacts that may be removed using compressed sensing (CS) [1]. CS undersampling for radial, stack-of-radials and stack-of-spirals has already been demonstrated [1-4]. For these acquisitions, the CS reconstruction is typically performed using conjugate-gradient (CG) type algorithms, where a gridding and regridding is performed at every iteration [1]. In this work, we investigate an alternative method for solving the CS objective function that does not require gridding and regridding at every iteration, reducing the computational complexity.

THEORY: The acquired non-Cartesian data is given by $\mathbf{s}=\mathbf{GFm}$, where $\mathbf{G}$ is the regridding operator, $\mathbf{F}$ is the Fourier transform and $\mathbf{m}$ is the image. Without loss of generality, we ignore de-apodization [5], since it can be corrected for the final image estimate. CS reconstruction solves an objective function of the form, $\arg\min_{\mathbf{m}} \frac{1}{2}\|\mathbf{s}-\mathbf{GFm}\|_2^2+\lambda\|\mathbf{\Psi}^*\mathbf{m}\|_1$, where $\mathbf{\Psi}$ is a unitary transform (e.g. image or wavelet domain). We first perform a quadratic relaxation [6] on the l_1 term by introducing an auxiliary variable $\mathbf{z}$ and change the objective function to: $\arg\min_{\mathbf{m,z}} \frac{1}{2}\|\mathbf{s}-\mathbf{GFm}\|_2^2+\lambda(\|\mathbf{z}\|_1+\beta/2\|\mathbf{z}-\mathbf{\Psi}^*\mathbf{m}\|_2^2)$. For large values of β, this enforces $\mathbf{z}$ to be close to $\mathbf{\Psi}^*\mathbf{m}$, reducing to the first problem. This is then solved using alternate minimization [7]: First $\mathbf{m}$ is kept fixed and $\mathbf{z}$ is updated by l_1 soft-thresholding coordinate-wise in $\mathbf{\Psi}$-domain. Then $\mathbf{z}$ is kept fixed and $\mathbf{m}$ is updated by solving: $\arg\min_{\mathbf{m}} \|\mathbf{s}-\mathbf{GFm}\|_2^2+\lambda\beta\|\mathbf{z}-\mathbf{\Psi}^*\mathbf{m}\|_2^2$, a least squares problem with solution, $\mathbf{m}_{est}=(\mathbf{F}^*\mathbf{G}^*\mathbf{GF}+\lambda\beta\mathbf{\Psi}\mathbf{\Psi}^*)^{-1}(\mathbf{F}^*\mathbf{G}^*\mathbf{s}+\lambda\beta\mathbf{\Psi}\mathbf{z})$. Noting that $\mathbf{\Psi}\mathbf{\Psi}^*=\mathbf{I}$, IFFT of $\mathbf{m}_{est}$ is $(\mathbf{G}^*\mathbf{G}+\lambda\beta\mathbf{I})^{-1}(\mathbf{G}^*\mathbf{s}+\lambda\beta\mathbf{F}(\mathbf{\Psi z}))$, which is computationally easier to solve. To avoid the inversion of the first term, we approximate $\mathbf{G}^*\mathbf{G}$ by a diagonal matrix, i.e. we find $\arg\min_{\mathbf{K}\text{ is diagonal}} \|\mathbf{K}-\mathbf{G}^*\mathbf{G}\|_F$, where $\|\cdot\|_F$ is the Frobenius matrix norm. Note $(\mathbf{K}+\lambda\beta\mathbf{I})$ is a diagonal matrix that can be inverted easily. Since gridding ($\mathbf{G}^*$) and regridding ($\mathbf{G}$) involve approximations, replacement of $\mathbf{G}^*\mathbf{G}$ by $\mathbf{K}$ is a reasonable approximation that allows us to avoid gridding and regridding at every iteration. We note similar approximations have been used in parallel imaging [8]. Furthermore, $\mathbf{K}$ also has an intuitive explanation: It is the result of re-gridding an all-ones k-space onto the spokes acquired and gridding these spokes back to a Cartesian k-space. In essence, $\mathbf{K}$ specifies the weights associated with a particular k-space location in the gridded data ($\mathbf{G}^*\mathbf{s}$) with points closer to the spokes or to the center getting a bigger weight, and it has to be calculated only once before reconstruction. Hence, the data-consistency step, $(\mathbf{K}+\lambda\beta\mathbf{I})^{-1}(\mathbf{G}^*\mathbf{s}+\lambda\beta\mathbf{F}(\mathbf{\Psi z}))$, provides a weighted average value of the acquired gridded k-space and the k-space corresponding to the thresholded estimate, normalized by the sum of weights for each (**Fig. 1**). In contrast, for the Cartesian case, the data consistency is typically done by replacing the acquired locations in the k-space of the thresholded estimate with the acquired lines [9], which is not possible in the non-Cartesian setting. Finally, we note other regularizers, such as total variation can be incorporated into the method easily, e.g. as in [7]. Thus iteration t of the proposed reconstruction algorithm is as follows: a) threshold current image estimate, $\mathbf{m}^{(t)}$ in $\mathbf{\Psi}$-domain, b) FFT the thresholded estimate (i.e. $\mathbf{F}(\mathbf{\Psi z})$) and perform data consistency, $(\mathbf{K}+\lambda\beta\mathbf{I})^{-1}(\mathbf{G}^*\mathbf{s}+\lambda\beta\mathbf{F}(\mathbf{\Psi z}))$, c) IFFT (b) to generate $\mathbf{m}^{(t+1)}$.

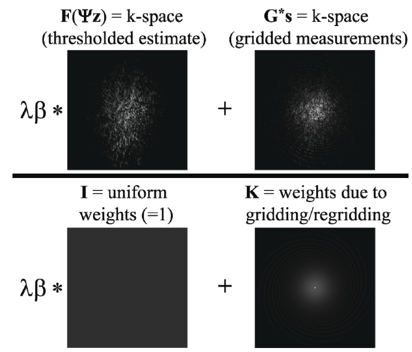

Fig. 1: *Data consistency in the proposed method for non-Cartesian acquisitions.*

MATERIALS AND METHODS: Whole heart ECG triggered, segmented cardiac MRI was acquired on a 1.5T Philips Achieva magnet with 5-channel cardiac coil. A 3D koosh-ball SSFP imaging sequence was used with the following parameters: TR/TE/α=4.2/2.1/60°, FOV=270×270×270mm^3,1.5×1.5×1.5mm^3. Proposed method was implemented in Matlab using Bregman method [6] for off-line reconstruction using image domain sparsity, where the corresponding de-apodization coefficients were used as thresholding weights for each voxel. The method was run for 20 iterations for each coil. Final estimate was generated by root-sum-squares of the coil images, followed by a region-of-interest based de-apodization. For comparison, data was also reconstructed with a traditional CG-type algorithm that does gridding, regridding and de-apodization at every iteration, which was implemented on a GPU, using image domain thresholding and 100 iterations. A reconstruction using the proposed method with TV regularization is also provided for comparison.

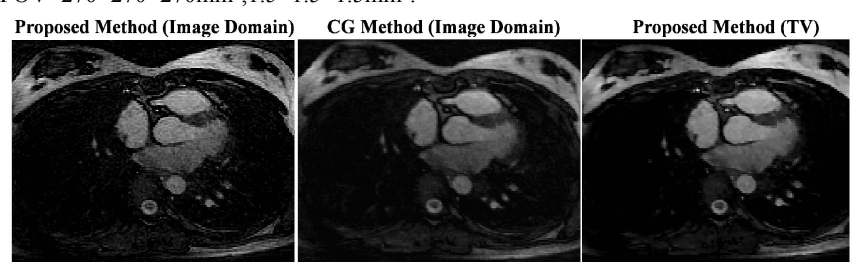

Fig. 2: *An example axial slice from a whole heart cardiac MRI dataset.*

RESULTS: **Fig. 2** depicts an axial slice from the reconstruction methods. The proposed reconstruction and CG reconstruction yield similar results, with the former requiring a small number of iterations. The proposed reconstruction in image domain results in sharper, albeit noisier images compared to the TV domain regularized implementation.

CONCLUSIONS: An iterative technique for CS reconstruction from non-Cartesian trajectories is proposed. This technique requires only two gridding and one regridding operation irrespective of the number of iterations, and has a fast empirical convergence rate.

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