

A New Robust Algorithm for Diffusion Tensor Evaluation

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Introduction

The diffusion tensor imaging toolbox exploits various diffusion maps based on scalar metrics such as mean diffusivity, fractional anisotropy (FA) and other rotational invariants. The quality of the maps relies on the accuracy of the diffusion tensor estimation from the raw experimental data [1]. In turn, estimation accuracy is influenced by the efficiency of the applied algorithms. The signal in DTI experiments is usually corrupted by noise. Additional artefacts may also originate from different physiological sources. As a consequence, diffusion tensor estimation becomes unstable and precarious, and may even give rise to negative definite tensors [2]. In order to resolve these problems, we have developed a new robust algorithm for diffusion tensor evaluation based on the non-linear least median squares (CNLMS). The developed algorithm has been tested using simulated data and *in vivo* experiments. The results are compared with the standard least squares algorithm and its constrained counterpart [2], as well as with the alternative robust evaluation algorithm, RESTORE, as proposed in Ref. [3].

Theory and Methods

In DTI experiments, the fitting objective can be formulated in several different ways. The most efficient one is a non-linear squared functional

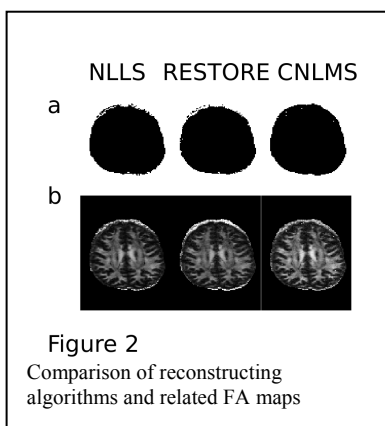
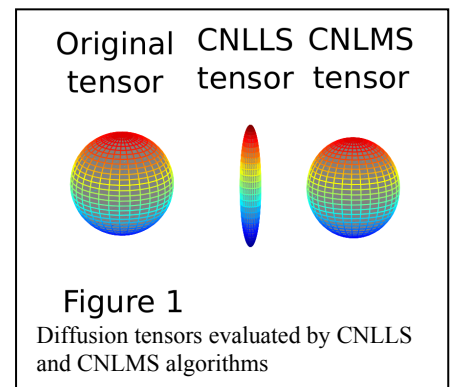
$$f_{NLLS}(\hat{D}) = \frac{1}{2} \sum_{i=1}^N [S_i - S_0 \exp(-bg_i^T \hat{D} g_i)]^2, \text{ where } \hat{D} \text{ is a diffusion tensor, } g_i \text{ is a strength of diffusion-encoding magnetic field gradients}$$

and N is a number of gradient directions. The minimisation objective of the developed robust algorithm is evaluated by replacing the sum of squared residuals in $f_{NLLS}(\hat{D})$ by the squared medians of the residuals, $f_{CNLMS}(\hat{D}) = \frac{1}{2} \sum_{i=1}^N \text{median}[S_i - S_0 \exp(-bg_i^T \hat{D} g_i)]^2$, where medians are

computed for all N directions. In order to guarantee the positive definite diffusion tensor we used an additional constraint based on the Sylvester criterion which was applied to both objectives, $f_{NLLS}(\hat{D})$ and $f_{CNLMS}(\hat{D})$. *In vivo* diffusion studies were carried out with a whole-body 3T Siemens MAGNETOM Tim-Trio scanner (Siemens Medical Systems, Erlangen, Germany). The range of diffusion weightings was [0, 200, 400, 600, 800, 1000] s mm⁻²; 6 non-collinear directions of the diffusion encoding gradients were applied. Numerical simulations were used to generate diffusion attenuations for the same gradient orientations as in the *in vivo* experiments and to reconstruct diffusion tensors. In numerical simulations, the original diffusion tensor was formed for the isotropic case with eigenvalues equal to [2.0; 2.0; 2.0] · 10⁻³ mm²s⁻¹. The simulated data were thereafter distorted by Gaussian noise and by additional outliers in all encoding directions with random amplitude. The outliers were placed at $b = 800$ s mm⁻².

Results and Discussion

Diffusion tensor estimation was performed using the developed method, CNLMS, and the constrained non-linear least squares (CNLLS) algorithm for a comparison. The diffusion tensor can be visualised in an ellipsoidal form in which the main axes of the ellipsoids are related to the eigenvalues of the diffusion tensor. The simulated original diffusion tensor takes the form of a sphere as in Figure 1. The results of tensor evaluation obtained by the CNLLS and CNLMS methods are also illustrated in Figure 1. The estimation from the CNLLS method yields an ellipsoid with a strongly pronounced anisotropic feature, [0.3±0.3; 1.4±0.3; 2.5±0.5] · 10⁻³ mm²s⁻¹. Unsatisfactory evaluation was shown to be due to the presence of the outliers. In contrast, the CNLMS method was capable of solving the problem with outliers much better and estimated ellipsoid has a nearly spherical form [1.4±0.1; 1.8±0.1; 2.0±0.1] · 10⁻³ mm²s⁻¹. Another application is demonstrated with *in vivo* experiments which allow us to elucidate the principle difference between the three applied algorithms: non-linear least squares (NLLS), RESTORE, and CNLMS. The results of the comparison are presented in Figure 2. The general problem of the unconstrained algorithms is that they tend to give rise to the negative eigenvalues. In Figure 2a, the voxels in which the negative solutions were encountered by the NLLS and RESTORE algorithms are marked in white (the rest of the voxels are represented in black). Most of the distortions were observed at the upper edge of the maps. As a consequence, the negative eigenvalues cause distortions in the FA maps (see Figure 2b for NLLS and RESTORE columns). The CNLMS method solves the problem of the negative eigenvalues. However, the trivial solutions for which the eigenvalues are zeroes cannot be completely avoided. In Figure 2a such voxels are shown in white (see the CNLMS column); they are clearly visible at the upper edge of the FA map in Figure 2b. A comparison of the FA maps for the CNLMS with that of the RESTORE shows that both the amount and the intensity of the distortions are significantly reduced in the former case. It is also noteworthy that the FA map in the case of the CNLMS produces a better



contrast than the other two methods.

Conclusion

The new algorithm for diffusion tensor reconstruction based on the least median squares is quite robust to the outliers and stable with respect to the noise variation. These features of the algorithm allowed us to obtain much more reliable values of the evaluated diffusion tensors in comparison to other methods. We believe that the robust algorithms of the tensor evaluation have a potential to find valuable applications in the research and in clinical practice.

References

- [1] P.J. Basser, J. Mattiello, D. Le Bihan, MR diffusion tensor and imaging, *Biophys. J.* 66 (1994) 259-267. [2] C.G. Koay, J.D. Carew, A.L. Alexander, P.J. Basser, M.E. Meyerand, Investigation of anomalous estimates of tensor-derived quantities in diffusion tensor imaging, *Magn. Reson. Med.* 55 (2006) 930-936. [3] L.C. Chang, D.K. Jones, C. Pierpaoli, *Magn. Reson. Med.* 53 (2005) 1088-1095.