Generalizing Diffusion Tensor Model using Probabilistic Inference in Markov Random Fields

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Introduction: We present initial results from *maximum a posteriori* (MAP) estimates of the number of tensors for each voxel in given DWIs. We model the prior distribution of tensor configurations with a *Markov Random Field* (MRF) that allows the expression of local conditional dependencies intuitively [1]. It is well-known that the standard single second-order tensor model of diffusion falls short of quantifying fiber geometries such as crossing (X), kissing (><), and branching (-<). This limitation has led to the development of different acquisition techniques along with more complex models of diffusion. One strategy to characterize the underlying complex fiber architecture is to quantify the diffusion function using the Fourier between the diffusion function and the diffusion signal attenuation in q-space [2]. Q-space methods, however, require a large number of scans (e.g., more than 100), incurring long acquisition times, which make them impractical in a clinical setting. Therefore, earlier work has proposed multi-tensor as well as simpler, restricted geometric model [e.g., 6,7]. We build on these earlier works.

Given signal values (i.e., DWI sequences) and a set of prior constraints, we would like to know the most likely tensor configuration that "explains" the signal. This is a typical Bayesian inference problem. Therefore, we model the probability distribution of configurations with a Markov Random Field (MRF) and obtain the most likely configuration using the MAP estimate.

MRFs: An MRF is a conditional probability distribution with a Markov property over a set of random variables described by an undirected graphical model $G(V,E)$ with a set V of vertices and a set E of edges [1]. The vertex set V of the graph corresponds to the random variables and the edge set E determines the conditional dependencies (i.e., Markov properties). One of the advantages of MRF is that it is easy to model local dependencies (or interactions) with arbitrary energy functions while ensuring the probability distribution stays as a proper probability distribution (The *Hammersley-Clifford* theorem ensures all MRFs have *Gibbs* representations, given $P(x) > 0$).

MAP estimate: Bayes's rule suggests that the posterior distribution $P(x|y) \propto P(y\,|x)P(x)$. In this context, the maximum a posteriori (MAP) estimate of an MRF is a choice of x that maximizes $P(x|y)$; In other words, it is the most likely assignment x of the random variables (i.e., nodes) given the observation y.

Energy formulation: MAP estimate in MRFs has an equivalent energy formulation [2] that we have used in our experiments. Let $V = \{v_1, v_2, \ldots\}$ be a set of random variables (e.g., corresponding the voxels in a DWI sequence) and L be a finite set of labels (e.g., number of tensors to fit). Let x_i denote a labeling of v_i; then $E(\boldsymbol{x}) = \sum_{\{v_i \in V\}} D(x_i) + \sum_{\{(i,j)\in N\}} W(x_i, x_j)$, where $D(x_i)$ is the cost assigning label x_i to v_i and $W(x_i, x_j)$ is the cost of assigning label x_i and x_j to two neighboring v_i andv_j. Minimizing $E(\boldsymbol{x})$ means finding a labeling assignment vector $\boldsymbol{x}$ that minimizes the cost.

Results: We ran three MAP estimate algorithms on a synthetic fiber-crossing data set, generated using b = 1500 and 12 gradient directions [5]. The algorithms experimented with are *iterated conditional mode* (ICM), *belief propagation* (BP), *and Tree-Reweighted Message Passing* (TWF) [4]. All the three algorithms, even the one as simple as ICM, provide good results, effectively selecting the configuration that represents the fiber crossing accurately (See Figures; ICM and TWF results are not shown due to space limitations).

Conclusions: We provided a proof of concept for efficiently modeling tensor configuration distributions with MRFs and their practical MAP estimations. The power of the MAP-MRF framework comes from its mathematical convenience in modeling prior distributions and the fact that it results in a global optimization driven by local neighborhood interactions (context). Note that it is very easy to incorporate a noise (degradation) factor into our model. We would like to test our ideas on synthetic data sets with different noise levels as well as real datasets in the future.

References: [1] Geman et al., Markov Random Fields and Their Applications to Computer Vision (1986). [2] Tuch, D. Q-ball imaging (2004) [3] Efficient Belief Propagation for Early Vision (2006). [4] Szeleski et al., A Comparative Study of Energy Minimization Methods for MRFs (2008). [5] Barmpoutis et al. Adaptive Kernels for Multi-fiber Reconstruction (2009). [6] Alexander, Multiple Fiber Reconstruction Algorithms for Diffusion MRI (2005). [7] Peled et al., Geometrically Constrained Two-tensor Model for Crossing Tracts in DWI (2006).

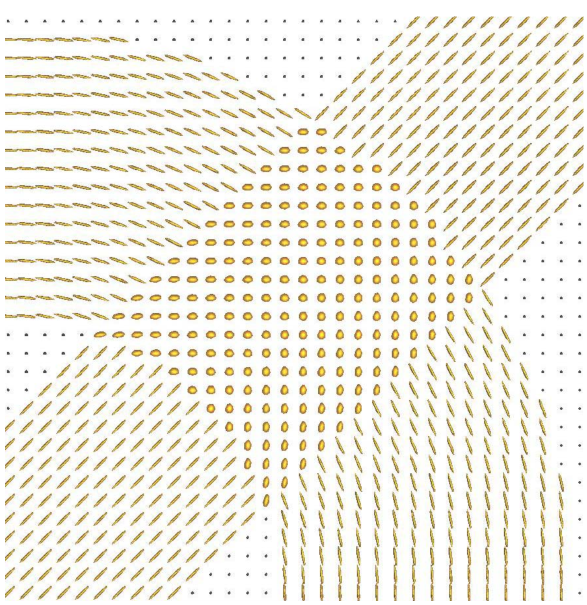

Figure 1 Single tensor fitting

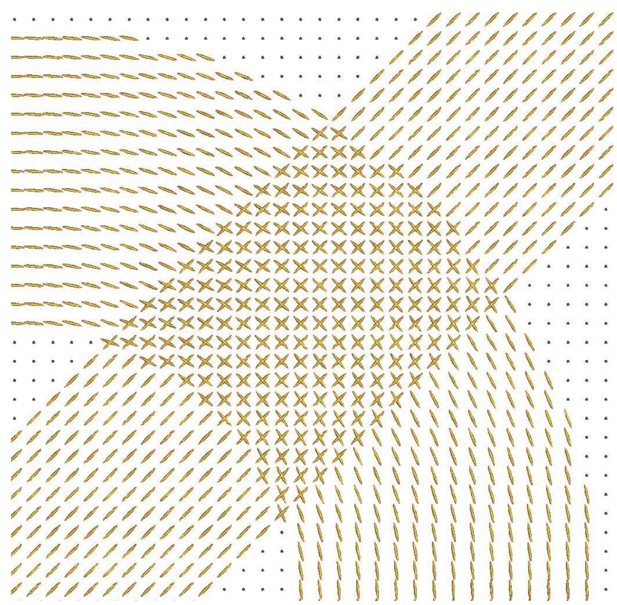

Figure 2 MAP Estimate with *belief propagation*