MULTI-SHELLED Q-BALL IMAGING WITHOUT ASSUMING INVERSION SYMMETRY

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Introduction: In diffusion MR analyses, the inversion symmetry of the water molecule diffusion is often assumed: the property of diffusion into a direction is identical to that into the opposite direction. Until now, generalized diffusion tensor imaging has been proposed as the method that can detect the inversion asymmetry (1). The diffusion will often be inversion asymmetric in the strict sense since the microstructure of a biological tissue is often inversion asymmetric. When the voxel size is much larger than the diffusion displacement and the scale of microstructures, the inversion asymmetry can be neglected (2,1). However, the technology that makes voxel size small to dozens of micrometers has been developed (3). Therefore, it is important to explore the method that detects the inversion asymmetry of the diffusion.

Recently, a novel method, multi-shelled q-ball imaging (MS-QBI), has been proposed (4). MS-QBI calculates the orientation distribution function (ODF) based on the moment of the probability density function (PDF) of diffusion displacement. The moment-based ODF can detect crossing fiber orientations more accurately than the ODF of conventional QBI (5) without increasing the sampling number. In this study, we propose a method to detect the inversion asymmetry of diffusion with MS-QBI. We also perform the numerical simulation of detecting the asymmetry and examine the ability.

Theory: The Fourier transform (FT) relation between PDF $P(\vec{R})$ and the normalized complex MR signal $S(\vec{q})$ is $S(\vec{q}) = \int_{-\infty}^{\infty} P(\vec{R})\, e^{i\vec{q}\cdot\vec{R}} d^3R \;\cdots$ [1] , where $\vec{q}$ is the q-vector and $\vec{R}$ is the diffusion displacement. In the cylindrical coordinate Eq. [1] becomes $S(q_\rho, q_\theta, q_z) = \int_0^\infty dR_\rho \int_0^{2\pi} dR_\theta \int_{-\infty}^{\infty} dR_z R_\rho P(R_\rho, R_\theta, R_z) \exp\left[i\{q_\rho R_\rho \cos(R_\theta - q_\theta) + q_z R_z\}\right] \cdots$ [2], where the subscripts ρ and θ denote the vector components along the radius and azimuthal directions, respectively. In MS-QBI we take the n-th derivative of Eq. [2] with respect to q_z. Then, by setting $q_z = 0$ and $q_\rho = q$ (q-ball radius) and integrating out with respect to q_θ, we have $\left(\frac{1}{i}\frac{\partial}{\partial q_z}\right)^n \int_0^{2\pi} S(q, q_\theta, q_z) dq_\theta \Big|_{q_z=0} = \int_0^\infty dR_\rho \int_0^{2\pi} dR_\theta \int_{-\infty}^{\infty} dR_z R_\rho J_0(qR_\rho)\, P(R_\rho, R_\theta, R_z) R_z^n \;\cdots$ [3] , where $J_0(\cdot)$ denotes the zero-th order Bessel function. The quantity of the right-hand side of Eq. [3] is a weighted average of R_z^n , which is a type of n-th order moment of PDF with respect to R_z ; this is the ODF value in z-direction of MS-QBI. This quantity is obtained as the left-hand side of Eq. [3], which can be known from the behavior of $\int_0^{2\pi} S(q, q_\theta, q_z) dq_\theta$ when q_z is varied near zero. If the inversion symmetry holds, odd order moments are zero. Thus an odd order moment can be used as the index that indicates the inversion asymmetry. In this study we calculate the ODFs of the first and second-order moments. For the calculations we return to Eq. [1] again. Note that the normalized signal $S(\vec{q})$ is a complex number in general. Since $P(\vec{R})$ is a real number, the FT relation means $S^*(\vec{q}) = S(-\vec{q})$, where the asterisk denotes the complex conjugate. Thus we have $\mathrm{Re}\{S(\vec{q})\} = \mathrm{Re}\{S(-\vec{q})\}$, $\mathrm{Im}\{S(\vec{q})\} = -\mathrm{Im}\{S(-\vec{q})\} \;\cdots$ [4] . Using Eqs. [4], we approximate and reduce the left-hand side of Eq. [3] for $n=1$ as $\frac{1}{i\Delta q_z}\int_0^{2\pi}\left\{S(q, q_\theta, \frac{\Delta q_z}{2}) - S(q, q_\theta, -\frac{\Delta q_z}{2})\right\} dq_\theta = \frac{2}{\Delta q_z}\int_0^{2\pi} \mathrm{Im}\{S(q, q_\theta, \frac{\Delta q_z}{2})\}\, dq_\theta \;\cdots$ [5] , where Δq_z is a parameter. The quantity of the right-hand side of Eq. [5] is the ODF value of the first-order moment. In the same way, we can approximate and reduce the left-hand side of Eq. [3] for $n=2$ as $-\frac{1}{\Delta q_z^2}\int_0^{2\pi}\left[\mathrm{Re}\{S(q, q_\theta, \Delta q_z)\} - \mathrm{Re}\{S(q, q_\theta, 0)\}\right] dq_\theta \;\cdots$ [6]. The quantity of Eq. [6] is the ODF value of the second-order moment. By setting the value of the parameter Δq_z in Eq. [5] to the double of that in Eq. [6], we can calculate the ODFs of the first and second-order moments using the common signal data sampled on the two spherical q-shells. Figure 1 shows the shell structure. The ODF value of the first-order moment in the thick red arrow-direction is proportional to the sums of the imaginary signals with respect to the thin red q-vectors. Since the ODF value of the first-order moment in the thick blue arrow-direction becomes -1 times of that of the thick red arrow-direction, we should calculate only either one. The ODF value of the second-order moment in the thick red arrow-direction is proportional to the difference between the sums of the real signals with respect to the thin red and thin black q-vectors. Since the ODF values of the second-order moment in the thick red and thick blue arrow are identical, we should calculate only either one.

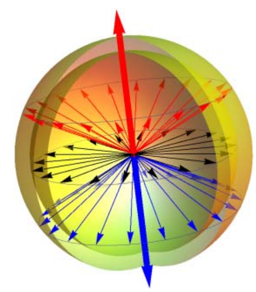

Fig. 1: q-shell structure

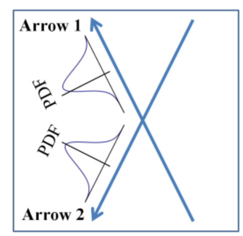

Fig. 2: Diffusion situation in a voxel (anisotropic and inversion asymmetric)

Simulation: Figure 2 shows the diffusion situation in a single voxel being our simulation subject. We use a simple model in which the diffusion is anisotropic and inversion asymmetric. The diffusion along two arrows are fast (diffusion eigenvalue is $\lambda = 2\times10^{-3}$ mm²/s) and those along the perpendicular directions are slow (diffusion eigenvalue is $\lambda/10$). PDFs along the arrows are non-zero mean Gaussian: the mean value along the arrow 1 is $\sigma/2$ (large asymmetry), and that along the arrow 2 is $\sigma/10$ (small asymmetry), where $\sigma = \sqrt{2\lambda(\Delta - \delta/3)}$ is the standard deviation of PDF. The diffusions along the perpendicular directions are inversion symmetric. Conditions for MS-QBI are as follows: q-shell radii are $b = 3800$ and 4200 s/mm². $\delta/\Delta = 20/43$ ms. Gradient direction number is 126 (hemisphere scheme). SNR is 4.0.

Figure 3 and 4 show the results of ODF profiles of the second and first-order moment, respectively. In Fig. 4, the red and blue colors indicate positive and negative values of the first-order moment, respectively. Figure 4 shows that the large inversion asymmetry along the arrow 1 can be detected but the small inversion asymmetry along the arrow 2 cannot be detected.

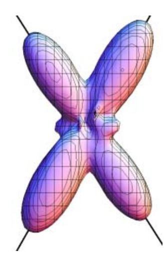

Fig. 3: Second-order moment ODF profile (an example)

Conclusion and discussion: MS-QBI can detect the inversion asymmetry of diffusion if the asymmetry is large enough. In our simple drift-diffusion model the asymmetry can be detected if the mean value of the PDF is around the mean diffusion displacement σ.

It is interesting to investigate the inversion asymmetry of the diffusion in biological system. For example, if a neuro-fiber bundle branches away in two directions in a voxel, the diffusion becomes inversion asymmetric. It is possible that the first-order moment ODF can detect the fiber branching with the inversion asymmetry in the case of the small voxel size realized by a high sensitivity coil system such as the cryoprobe (3).

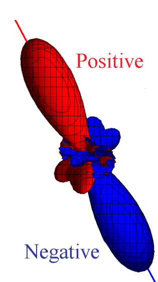

Fig. 4: First-order moment ODF profile (an example)

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