Sparsity Characterisation of the Diffusion Propagator

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INTRODUCTION: Compressed Sensing (CS) can be seen as a recipe with three ingredients: a sparse basis to represent the data, a maximally incoherent sensing/sampling basis and a nonlinear convex method that usually minimize the ℓ_1-norm and/or the total variation semi-norm [1]. Recent works have mostly focused on ingredients 1-2-3 in the HARDI/DTI (High Angular Resolution Diffusion Imaging/Diffusion Tensor Imaging) CS problem [2,3,4]. Here, we focus on the problem of finding the best sparse basis to represent the diffusion propagator in diffusion-weighted imaging. Under the narrow pulse approximation, the diffusion propagator framework relates the diffusion signal and diffusion propagator through a Fourier relationship, $P(\mathbf{r}) = F[S(\mathbf{q})/S_0]$, where F is the Fourier transform operator, $S(\mathbf{q})$ is the Diffusion Spectrum Imaging (DSI) data [5] and S_0 is the b-value = 0 s/mm^2 signal. Since the propagator itself can be viewed as a 3-dimensional (3D) image, we choose to attack to problem with the state-of-the-art wavelets armada [6] extended in 3D. Note that a preliminary study [7] used a Daubechies-8 (D8) wavelet basis to show the potential of CS for DSI. Related to the second ingredient, [8] has explored different sampling strategies to accelerate DSI. Here, we executed a battery of experiment on both synthetic and human brain data to answer these specific questions: What is the best sparsifying wavelet transform for the diffusion propagator representation? Are the biorthogonal wavelet transforms showing improvements over standard orthogonal transforms? What is the best approximation scheme for optimal quality of the sparse diffusion propagator reconstruction; hard and soft thresholding?

METHODS: We created various synthetic DSI signals based on real-data schemes of [5]. We generated 515 direction vectors sampled on the Cartesian lattice inside a ball of radius 5 and a maximum b-value of 6000 s/mm^2. We also used 1-fiber, 2-fibers crossing at angles of 45°, 60° and 90°, 3-fibers crossing at 90° signals with respective equal volume compartments having Fractional Anisotropy (FA) of 0.8. We also contaminated the signals with a Rician noise level of $\sigma = 0.01, 0.02, 0.03333$, to obtain SNR = 100, 50, and 33 in the baseline image S_0. The 3D wavelets transforms implemented are: Haar, D4, D8, CDF 5-3, CDF 9-7, where D stands for Daubechies and CDF stands for Cohen-Daubechies-Fauveau [6]. We also performed two nonlinear thresholding methods to sparsify the diffusion propagators: namely hard and soft thresholding [6]. For our similarity and quality metrics, we used SNR, relative error and Kullback-Leibler divergence (KL-div) to quantify the resemblance of the reconstructed sparse diffusion propagator with the original noiseless diffusion propagator. We also exhaustively produced 3D visualization figures for a qualitative analysis. For the DSI human brain data, the acquisition was performed on a 3-Tesla Philips magnetic resonance system, max b-value = 6000 s/mm^2, 2 mm isotropic voxels, 515 gradient directions (same as our synthetic experiment) and a Echo Planar Imaging (EPI) sequence with parallel imaging with an acceleration factor of 2. The DSI acquisition time was 1h45 minutes. In this dataset, we chose several voxels of interest to reconstruct the diffusion propagator; a 1-fiber voxel in the corpus callosum, an isotropic voxel in the ventricules, and a 2-fiber crossing voxel between the corona radiata (CR) and the superior longitudinal fasciculus (SLF).

RESULTS: Tables 1 and 2 show the KL-div results for various synthetic DSI signal configurations. Table 1 highlights a simulation with Rician noise level of σ=0.01 and only 11% of the wavelets coefficients used in the propagator reconstruction. In this case, the best reconstructions are achieved with the biorthogonal CDF 9-7 basis and a nonlinear soft threshold. Similarly, Table 2 highlights a simulation with noise level of σ=0.02 and approximation with only 5% of the wavelets coefficients. In that case, the best reconstructions are achieved with the biorthogonal CDF 5-3 basis and a nonlinear soft threshold. Figure 1 shows a 3D diffusion propagator from a voxel in the CR-SLF section of a human brain. This visual representation is formed by isosurfaces using regular sampling intervals. Figure 1A shows the original diffusion propagator, 1B the approximation using a nonlinear soft threshold with 1% of the coefficients and, 1C, by 7% of the coefficients. In that case, 7% is enough to obtain an accurate approximation. Figure 2 shows the SNR and relative error curves obtained when comparing a noisy synthetic two-fibers bundle crossing at 90° and its perfect noiseless signal. Using the CDF 5-3, we can see that our best SNR and relative error are between 5% and 13% of coefficients, demonstrating the value of combining biorthogonal wavelets, nonlinear soft thresholding and the good percentage of coefficients in the diffusion propagator approximation.

KL-div	Haar		D4		D8		CDF 5-3		CDF 9-7	
Thresholding	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft	Hard	Soft
2 fibers 90°	0.09	0.12	0.08	0.08	0.08	0.08	0.09	**0.06**	0.08	**0.06**
2 fibers 45°	0.10	0.11	0.08	0.08	0.08	0.08	0.08	0.07	0.08	**0.06**
2 fibers 60°	0.10	0.11	0.08	0.08	0.08	0.08	0.08	**0.06**	0.08	**0.06**
3 fibers 90°	0.09	0.10	0.07	0.07	0.07	0.07	0.08	**0.06**	0.08	**0.06**

Table 1. KL-div from synthetic data, keeping 11% of coefficients, Rician noise: σ=0.01. Best results are obtained for CDF 9-7 in bold.

KL-div	Haar		D4		D8		CDF 5-3		CDF 9-7	
Thresholding	hard	soft	hard	soft	hard	soft	hard	soft	hard	soft
2 fibers 90°	0.25	0.26	0.20	0.20	0.20	0.20	0.17	**0.13**	0.18	0.14
2 fibers 45°	0.25	0.24	0.20	0.20	0.20	0.20	0.18	**0.15**	0.19	**0.15**
2 fibers 60°	0.24	0.22	0.18	0.18	0.18	0.18	0.17	**0.12**	0.17	0.14
3 fibers 90°	0.22	0.22	0.17	0.17	0.17	0.17	0.17	**0.13**	0.16	**0.13**

Table 2. KL-div from synthetic data, keeping 5% of coefficients, Rician noise: σ=0.02. Best results are obtained for CDF 5-3 in bold.

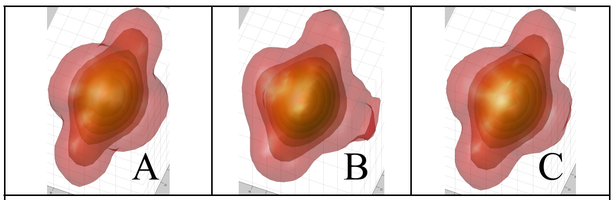

Figure 1. The diffusion propagator of a voxel of interest in the CR-SLF section of a human brain. A– Original diffusion propagator. B- Approximation based on a CDF 9-7 wavelets transform and nonlinear soft thresholding keeping 1% of the coefficients and C- keeping 7% of the coefficients. In that case, 7% is enough to obtain an accurate approximation.

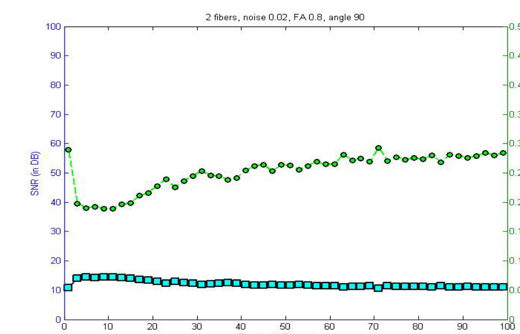

Figure 2. SNR (blue squares) and relative error (green circles) when comparing a noisy (rician noise level of : σ=0.02) synthetic two-fibers bundle crossing at 90° with the perfect noiseless signal. Here we used a CDF 5-3 wavelets basis with nonlinear soft thresholding. Note that the best performances are achieved between 5% and 13 % of coefficients.

DISCUSSION & CONCLUSIONS: Our recommendation for the best 3D wavelet basis to obtain a sparse diffusion propagator representation is the biorthogonal CDF wavelets with a nonlinear soft threshold. This is true from both a quantitative and qualitative point of view. As demonstrated in [6], the lifting scheme used for biorthogonal wavelet transform manages well the boundary effects and usually performs better than the periodic scheme used in orthogonal wavelet. The benefits are even more present when the Rician noise level increases, most importantly for high b-value images at the boundary of q-space. This is an important issue for DSI. Also, note that both D4 and D8 performed exactly the same way in the quantitative KL-div experiment, which suggests that the number of vanishing moments or compact support of wavelets have little effect on the sparse diffusion propagator representation. This is also seen in the quantitative comparison of CDF 5-3 and CDF 9-7. It also appears that biorthogonal wavelet transforms perform well to sparsify the diffusion propagator in a real human brain dataset. As in our synthetic experiments, our real data results show that approximately 7% of the wavelets coefficients of a CDF basis used in conjunction with a nonlinear soft thresholding are adequate to approximate all types of diffusion propagators. This is a promising result as 1st ingredient of a compressed sensing solution for DSI. **REFERENCES:** [1] Candès et al, IEEE Signal Process. Mag. 2008. [2] Michailovich & Rathi, MICCAI 2010. [3] B. Landman et al, ISMRM 2010. [4] Merlet & Deriche, MICCAI workshop. [5] Wedeen et al, MRM 2005. [6] Mallat, A wavelet tour of signal processing, 3rd edition, 2009. [7] Lee et al, ISMRM 2010. [8] Menzel et al, ISMRM 2010.