

Quantum Perspectives in Radiation Damping: Rabi Nutation and the Onset of Free Induction Decay

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Hoult's investigation (1) of the quantum origin of the free induction decay notwithstanding, most treatments of radiation damping are still given classically (2); this in despite of a vast literature (3) on the Jaynes-Cummings (J-C) model in quantum optics, for the coupling of a two level atom to a cavity. We here propose that the J-C calculation of Rabi nutation, of a two level atom coupled to a maser cavity, provides a computational platform applicable to spin $\frac{1}{2}$ nuclei in NMR, with the quantized LC circuit standing in for the cavity. In this picture, reception occurs by direct exchange of a virtual photon between spin and coil, which comports with views expressed earlier by Hoult and Ginsburg (1).

J-C considered a single polarized ammonia (4) molecule coupled to the lowest TM mode of cylindrical cavity (4) and obtained a Rabi frequency of 5 Hz, ~ 7 orders of magnitude faster than spontaneous emission in free space. Then a straightforward calculation (5, 6) gives the Rabi frequency for a proton coupled to a coil: $\Omega = \gamma\sqrt{2\hbar\omega_0L}/4a$. Here γ is the gyromagnetic ratio, $\sqrt{2\hbar\omega_0L}$ is the flux square per square root of the oscillator occupation number, and a is the coil aperture. For a Helmholtz coil of 115 nH inductance, i.e. a pair of loops of radius 0.65 cm, separated by ~ 0.7 cm, with zero leakage flux, we get $\Omega/2\pi = 2.4 \times 10^{-5}$ Hz for a single proton at 14.1 tesla (i.e. at a Larmor frequency of 600 MHz.) By contrast, the time constant for spontaneous emission at this field $\mu_0\hbar\gamma^2\omega^3/\pi c^3 = 1.5 \times 10^{-22}$ per second. The differential is a staggering 17 orders of magnitude. However, this enormous factor alone cannot account for the emitted power in a typical radiation damping experiment. Re-writing the energy balance equation (2) in terms of the Rabi frequency (immediately following a $\pi/2$ pulse) gives $dE/dt = M_0VB_0\Omega/2\pi$, which, scaled up for a sample of neat water, in a 5 mm NMR tube, predicts an emitted power of about 2×10^{-11} W. This is far below the required value of 1.2×10^{-5} W, for a fully developed damping current of about 5 mA into 2 ohms (source + load), as obtained by the Bloch-Kirchhoff (B-K) equations (7), cf Figure 1. Still, examination of the very early time course (Fig. 2) shows (at the outset) power levels consistent with the initial phase of the FID being powered only by Rabi nutation, prior to the onset of the full damping current.

A simple expression relates the peak current in the coil to the oscillator occupation number (ignoring the zero point energy): $\frac{1}{2}LI^2 = N\hbar\omega_0$. Then, a damping current of, say, 1 mA in a 115 nH coil enforces occupation numbers on the order of $N = 10^{11}$. This suggests an alternate view to that proposed by Bloembergen and Pound, in their classic paper of 1954 (8). There they argue from a semi-classical picture, that radiation damping is a form of coherent spontaneous emission, enhanced by the density of states inside a high Q resonator. The fully quantized picture based on the J-C model suggests the possibility of stimulated emission. That is, the direct exchange of virtual photons between spins and coils is directly enhanced by the large occupation number, and may therefore be viewed as stimulated emission. This conclusion is of course tentative, and will require further analysis and discussion.

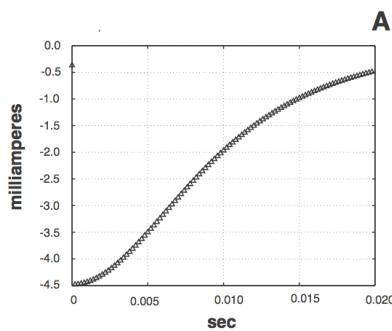


Fig. 1: A) Long term time course of radiation damping current in NMR receiver coil, from neat water sample at 600 MHz, calculated from B-K equations, with assumed p-90 of 8.5 us at 16 W, and a net probe resistance (source plus load) of 2 ohms. Note initial point near -0.5 mA, followed by precipitous drop to -4.5 mA. B) Expanded time view of early stage damping current, showing the region of the drop.

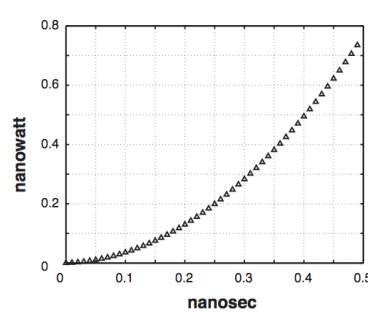


Fig. 2: Very early stage time course of emitted receiver power from probe, calculated by B-K equations with conditions the same as in Fig. 1. The initial power is the range 10 pW, consistent with Rabi nutation.

References:

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