Spatiotemporal Denoising of MR Spectroscopic Imaging Data by Low-rank Approximations

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INTRODUCTION

Low signal-to-noise ratio (SNR) has been a major limitation for magnetic resonance spectroscopic imaging (MRSI). A straightforward way to improve the SNR of MRSI data is to acquire more signal averages but at the expense of lengthening the already long data acquisition time. Several post-processing methods have been proposed to reduce noise in MRSI data, either by applying general denoising methods such as wavelet shrinkage or by imposing a specific parametric model for MRS signal [1-3]. This paper presents a novel denoising method, called LORA (LOw Rank Approximations), which exploits two low-rank properties of MRSI data: one due to partial separability and the other due to linear predictability of MRSI signals.

THEORY

The measured noise-corrupted MRSI data can be expressed as

$$\hat{s}(k,t) = s(k,t) + \xi(k,t),$$

where $s(\boldsymbol{k},t)$ represents the noiseless data and $\xi(\boldsymbol{k},t)$ is the measurement noise. We propose to denoise $\hat{s}(k,t)$ by exploiting the following low-rank structures:

a) Low-rankness due to partial separability: $s(\boldsymbol{k},t)$ can be approximated as partially-separable to the N-th order as

$$s(k,t) = \sum_{n=1}^{N} c_n(k)\phi_n(t). \qquad (1)$$

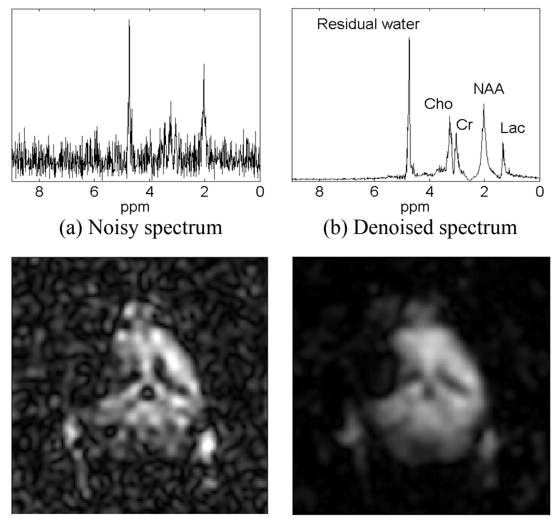
Expression (1) implies that the Casorati matrix $\boldsymbol{C}$ formed from $s(\boldsymbol{k}_n,t_m)$ (see equation (2)) has at most rank N [4]. In practice, N is much smaller than the size of the matrix.

$$C = \begin{bmatrix} s(k_1,t_1) & s(k_1,t_2) & \cdots & s(k_1,t_M) \\ s(k_2,t_1) & s(k_2,t_2) & \cdots & s(k_2,t_M) \\ \vdots & \ddots & & \vdots \\ s(k_{N_v},t_1) & s(k_{N_v},t_2) & \cdots & s(k_{N_v},t_M) \end{bmatrix}; \quad H = \begin{bmatrix} s(t_1) & s(t_2) & \cdots & s(t_K) \\ s(t_2) & s(t_3) & \cdots & s(t_{K+1}) \\ \vdots & \vdots & & \vdots \\ s(t_{M-K+1}) & s(t_{M-K+2}) & \cdots & s(t_M) \end{bmatrix} \qquad (2)$$

b) Low-rankness due to linear predictability: The ideal signal of a spin system with L spectral components resonating at frequency f_l with damping factor α_l can be expressed as

$$s(t_m) = \sum_{l=1}^{L} a_l e^{-(\alpha_l + j2\pi f_l)m}. \qquad (3)$$

In practice, non-Lorentzian resonances occur but can be well approximated by (3) with a large order L. It follows from (3) that the Hankel matrix $\boldsymbol{H}$ formed from $s(t_m)$ (see equation (2)) has rank L.



(a) Noisy spectrum (b) Denoised spectrum

(c) Noisy spatial distribution d) Denoised spatial distribution

Figure 1. Denoising results with in-vivo experimental data: (a),(b) Spectrum from a particular voxel (shown in absolute mode); (c),(d) Spatial distribution from 1.6 ppm to 2 ppm.

c) Algorithm: Exploiting above described low-rank properties of MRSI data, we can effectively denoise $\hat{s}(k,t)$ by finding an optimal rank-N approximation of $\boldsymbol{C}$ and optimal rank-L approximation of $\boldsymbol{H}$. This can be easily done with SVD. The specific algorithm is summarized below:

1. Given noisy data $\hat{s}(k_n,t_m)$, construct matrix $\boldsymbol{C}$ according to (2) and perform SVD truncation to obtain its N-th rank approximation $\boldsymbol{C}_N$.
2. Take the discrete Fourier transform along each column of $\boldsymbol{C}_N$ to obtain $\bar{s}(r_n,t_m)$.
3. For each voxel $\boldsymbol{r}_n$, form matrix $\boldsymbol{H}$ from $\bar{s}(r_n,t_m)$ and obtain its L-th rank approximation $\boldsymbol{H}_L$.
4. The denoised data is then obtained by extracting the elements from the first row and last column of $\boldsymbol{H}_L$.

METHODS

We applied the proposed denoising method to in-vivo MRSI data of the mouse brain, which was previously used in [5]. The CSI data was acquired using Varian INOVA 11.74 T (500 MHz) MRI system, TE=270 ms, TR=1500 ms, bandwidth=6000 Hz, 1024 FID data points for each of 32×32 k-space samples, and 8 averages. A detailed description of the dataset can be found in [5].

RESULTS

Figure 1 shows a typical set of results. It is clear that with LORA, noise has been effectively suppressed while spatial and spectral features were preserved. In particular, the denoised spectrum at a particular voxel shown in Fig. 1(b) clearly shows the resonances of residual water and metabolite peaks with reduced noise variance; the denoised spatial distribution obtained by integrating the complex spectrum from 1.6 ppm to 2 ppm has well-preserved spatial features in the electrocogualation and CSF regions.

CONCLUSIONS

A new method for spatial-spectral denoising of MRSI data has been presented. The method exploits two low-rank properties in MRSI data, one due to partial-separability and the other due to linear predictability of MRSI data. Experimental results from practical data demonstrate that the method is effective in reducing noise in MRSI data in a wide range of SNR values.

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