# A General $T_{1\rho}$ Relaxation Model for Spin-Lock MRI using a Rotary Echo Pulse

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**Introduction:** $T_{1\rho}$ relaxation is conventionally described as a purely mono-exponential decay with respect to the spin lock (SL) time (TSL). However, $B_0$ and $B_1$ inhomogeneities can be significant sources of SL artifacts in $T_{1\rho}$-weighted imaging and violate the validity of the mono-exponential decay [1]. As such, considerable error may be resulted by following the mono-exponential decay for $T_{1\rho}$ mapping, particularly at low SL frequencies (FSL). We present a general theoretical $T_{1\rho}$ relaxation model using a rotary echo SL pulse [2]. An analytical expression of $T_{1\rho}$-weighted longitudinal magnetization was derived to show its dependence on off-resonant frequencies, TSL, FSL, $T_{1\rho}$ and $T_{2\rho}$ relaxation time.

**Theory:** A SL rotary echo pulse with FSL $\omega_1$ was shown in Fig. 1. The magnetization evolution can be traced by using the Bloch Equation. An RF pulse is represented in the form of matrix notation $R_{\varphi}(\Phi)$, where $R$ denotes a rotation matrix, $\varphi$ is the pulse orientation and $\Phi$ is the pulse flip angle ($\Phi=\omega_1 \cdot TSL/2=2\pi \cdot FSL \cdot TSL/2$). The full expression of the magnetization evolution in the presence of off-resonance $\Delta\omega_0=2\pi\Delta f_0$ is expressed as

$$M(t) = R_{-x}(90^\circ)\cdot R_{z'}(\alpha)E_\rho \cdot R_{z'}(\alpha)E_\rho \cdot R_x(90^\circ)\cdot M(t_0) = R_{-x}(90^\circ)\cdot R_x(-\theta)R_z(\alpha)E_\rho R_x(\theta)\cdot R_x(\theta)R_z(\alpha)E_\rho R_x(-\theta)\cdot R_x(90^\circ)\cdot M(t_0) \quad (1)$$

where $\alpha=\omega_{eff}\cdot TSL/2$, $\theta=\tan^{-1}(\omega_1/\Delta\omega_0)$ and $\omega_{eff}=(\omega_1^2+\Delta\omega_0^2)^{1/2}$. $E_\rho$ is a matrix to describe $T_{1\rho}$ and $T_{2\rho}$ relaxation as shown in Eq.2.

$$E_\rho = \begin{bmatrix} e^{-TSL/2T_{2\rho}} & 0 & 0 \\ 0 & e^{-TSL/2T_{2\rho}} & 0 \\ 0 & 0 & e^{-TSL/2T_{1\rho}} \end{bmatrix} = \begin{bmatrix} E_{2\rho} & 0 & 0 \\ 0 & E_{2\rho} & 0 \\ 0 & 0 & E_{1\rho} \end{bmatrix} \quad (2)$$

Finally, the resultant longitudinal magnetization after the SL pulse excitation can be expressed by Eq.3.

$$M_z(t) = (E_{2\rho}^2\cos^2\alpha\cos^2\theta\cos 2\theta - E_{2\rho}^2\sin^2\alpha\cos^2\theta + E_{1\rho}E_{2\rho}\cos\alpha\sin^2 2\theta - E_{1\rho}^2\sin^2\theta\cos 2\theta)M_0 \quad (3)$$

**Methods:** A numerical program was developed to simulate the magnetization Mz as a function of $T_{1\rho}$, $T_{2\rho}$ and $\Delta\omega$ at different FSLs. Spin lock imaging on a phantom ($NiCl_2$ $6H_2O$) and rat livers was performed on a 3T Philips Achieva scanner using a 3D balanced FFE sequence (TR/TE=5.0/2.5ms, FA=30º, pixel size =0.35mm, thickness=2mm) with different FSLs. The delay time after acquisition was set as 4000ms to restore the equilibrium magnetization prior to the next $T_{1\rho}$ preparation.

**Results:** The magnetization $M_z$ as a function of $T_{1\rho}$, $T_{2\rho}$, TSL and $\Delta\omega_0$ at different FSLs is shown in Fig. 2. $M_z$ shows an attenuated oscillating decay instead of a mono-exponential decay as described by the conventional model. With FSL>> $\Delta\omega_0$, the model can be simplified as the conventional

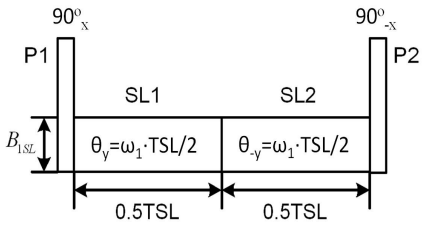


Fig.1. The pulse structure of a SL rotary echo

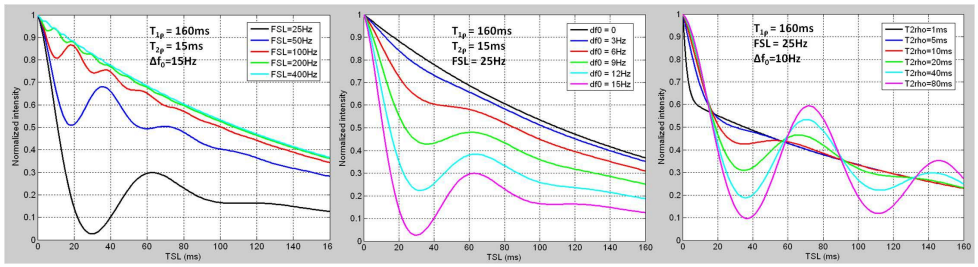


Fig.2. Simulation of longitudinal magnetization as a function of $T_{1\rho}$, $T_{2\rho}$ and $\Delta\omega_0$ at different FSLs

model of mono-exponential decay with TSL. It is seen in Fig.2 that FSL determines the oscillating frequency, $\Delta\omega_0$ determines oscillating amplitude and $T_{2\rho}$ determines oscillating decay rate. $T_{1\rho}$ magnetization decay curves (solid lines) and the fitting curves (dash lines) based on Eq. 3 in phantom and rat liver at FSLs are shown in Fig. 3. Excellent goodness of fit with high coefficient of determination ($R^2$) over 0.92 was achieved by using the general $T_{1\rho}$ relaxation model for all FSLs except that at 25Hz.

**Discussion:** As indicated by the general model, the conventional mono-exponential decay model cannot be used for $T_{1\rho}$ mapping at low FSLs because $T_{2\rho}$ and $\Delta\omega_0$ have significant influence on $T_{1\rho}$ weighting. The proposed general is helpful for $T_{1\rho}$ mapping particularly at low FSLs, benefiting patients with much lower RF energy deposit and lower SAR. Another advantage of this general model is that it has potential for simultaneous multi-parameter mapping of T2ρ and off-resonance additional to the normal T1ρ mapping.

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**References:** [1] Witschey WR et al, JMR 186:75-85(2007); [2] Charagundla SR et al, JMR, 162:113-21(2003);

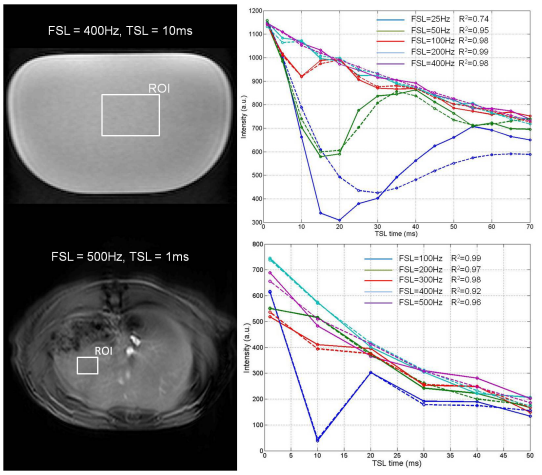


Fig.3. Experimental verification of $T_{1\rho}$ relaxation at different FSLs