MR Flow Imaging Beyond the Mean Velocity: Estimation of the Skew and Kurtosis of Intravoxel Velocity Distributions

P. DYVERFELDT[1,2], A. SIGFRIDSSON[1], H. KNUTSSON[1], AND T. EBBERS[1]
[1]CMIV AND LINKÖPING UNIVERSITY, LINKÖPING, SWEDEN, [2]UNIVERSITY OF CALIFORNIA SAN FRANCISCO, SAN FRANCISCO, CA, UNITED STATES

INTRODUCTION: MR flow imaging continuously improves the understanding and diagnostics of cardiovascular disease [1]. The most common MR flow imaging method is phase-contrast MRI (PC-MRI) velocity mapping, which estimates the mean velocity of a voxel. However, MR has the ability to do much more advanced flow analysis, including quantification of turbulent flow [2, 3]. The aim of this work was to go beyond the mean velocity of a voxel and demonstrate the potential of MRI for the quantification of the skew and kurtosis of intravoxel velocity distributions. Skew describes the asymmetry of a velocity distribution and kurtosis describes the sharpness of its central peak.

THEORY: The MRI signal $S(k_v)$, where $k_v = \pi$/VENC is the motion sensitivity, is the Fourier transform of the intravoxel spin velocity distribution s(v). Moments (see table 1) in the function domain (velocity-space) correspond to differentials in the Fourier transform domain (k_v-space) at $k_v = 0$ [4]. Thus, MRI has the potential to quantify any moment of a velocity distribution [3]. The skew and kurtosis can be obtained from the third and fourth central moments of the distribution, respectively (table 1). This require determination of derivatives of the MRI signal $S(k_v)$ at $k_v = 0$ up to the 3rd and 4th order, respectively.

Table 1. Examples of moments of a velocity distribution

Moment	Interpretation
First raw moment, μ'_1	Mean = μ'_1
Second central moment, μ_2	Standard deviation = sqrt(μ_2)
Third central moment, μ_3	Skew = μ_3/σ^3
Fourth central moment, μ_4	Kurtosis = μ_4/σ^4

METHODS: To demonstrate proof-of-concept, estimations of the skew and kurtosis were performed based on finite difference approximations of the derivatives of $S(k_v)$ using a tailored MRI simulation approach [2]. Realistic velocity distributions with known skew and kurtosis were obtained by extracting isotropic voxels from numerical velocity fields of post-stenotic flow generated by large-eddy simulations [5]. For each voxel, the intravoxel velocity distribution s(v) was obtained by computing a probability density estimate of the velocities of the virtual spins. From s(v) and the velocities of the virtual spins, $S(k_v)$ was computed for a range of k_v values. The 1st to 4th derivatives of $S(k_v)$ at $k_v = 0$ were estimated by finite differentiation, both in absence and presence of noise. Results on skew and kurtosis were evaluated as functions of the spacing, Δk_v, used in the finite differentiation. Note that while small Δk_v is preferable for the derivative estimation, noise may necessitate the use of a larger Δk_v in practice.

RESULTS AND DISCUSSION: Two intravoxel velocity distributions with different skew and kurtosis are shown in the upper row of Fig. 1. The true values for skew and kurtosis of these distributions are indicated by the dashed lines in the middle and bottom rows of Fig. 1. Evidently, finite difference approximations of the derivatives of $S(k_v)$ permit accurate determination of skew and kurtosis for small values of Δk_v (high VENC) in absence of noise. As expected, however, the accuracy of these finite difference approximations decline as the spacing Δk_v increases.

As small differences in motion sensitivity implies small differences in $S(k_v)$, noise severely degrades the finite difference estimation when Δk_v, is small (see Fig. 1). Improved robustness to noise may be obtained by modeling specific aspects of the velocity distribution. In PC-MRI velocity mapping, robustness to noise is improved by assuming symmetric velocity distributions [6]. Similarly, when $s(v)$ can be approximated by a specific distribution, such as a Gaussian, robust estimates of the standard deviation of $s(v)$ can be obtained [2]. These assumptions introduce model errors but significantly decrease the noise sensitivity [3].

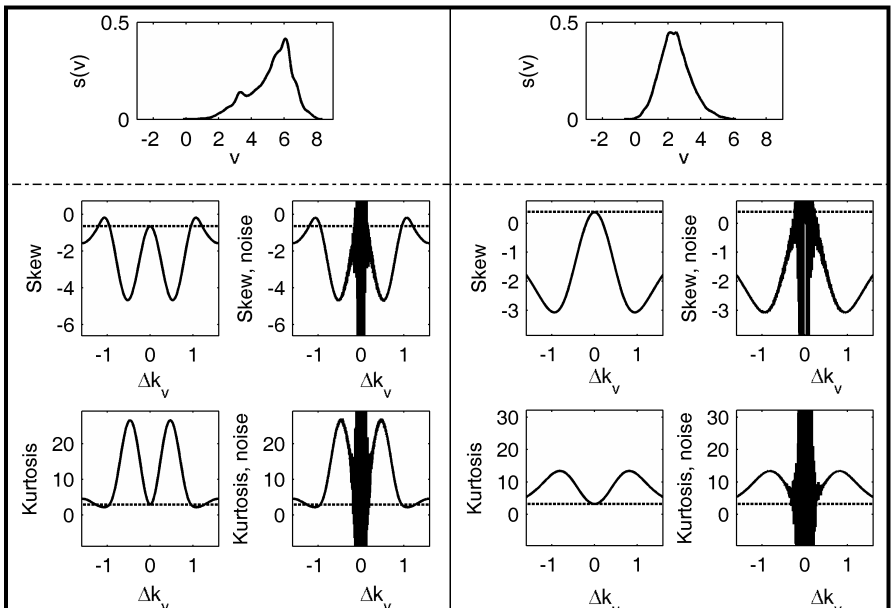

Figure 4. Upper row: Two different intravoxel velocity distributions. Middle row: The skew of the distributions estimated in absence and presence of noise. Bottom row: The kurtosis of the distributions estimated in absence and presence of noise. In each subpanel, the dashed lines indicate the true values.

CONCLUSION: MRI has the potential to quantify the skew and kurtosis of intravoxel velocity distributions. The finite difference approach used here for proof-of-concept requires very high SNR, which may be unattainable for scan times feasible *in vivo*. However, by modeling specific aspects of $s(v)$, as done in PC-MRI velocity and intravoxel velocity standard deviation mapping, more robust approaches to the estimation of skew and kurtosis may be derived. Being a measure of the asymmetry of velocity distributions, measurements of skew may help in assessing the significance of the model errors made in conventional PC-MRI velocity mapping.

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