Total Variation Denoising with Spatially Dependent Regularization

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Introduction: The Total Variation (TV) regularization model is popular in MR research for various applications including denoising [1] or constrained image reconstruction [2]. In the TV-model, a regularization parameter controls the trade-off between noise elimination, and preservation of image details. However, MR images are comprised of multiple details. This indicates that different amounts of regularization are desirable for regions with fine image details in order to obtain better restoration results. In this work spatially dependent regularization parameter selection for TV based image restoration is introduced. Utilizing this technique, the regularization parameter is adapted automatically based on the details in the images, which improves the reconstruction of details while still providing adequate smoothing for the homogeneous parts.

Theory: In order to enhance image regions containing details while still sufficiently smoothing homogeneous parts, we improve the TV-model by using a spatially dependent regularization parameter instead of a scalar value only, i.e. we consider,

$$\min_u \; \frac{1}{2}\int_\Omega \lambda(x)\,|u-z|^2\; dx + TV(u), \qquad (1)$$

where z is the noisy image, u is the restored image, and $TV(u)$ is the conventional TV penalty term [1]. In this model, λ is localized at image features. For small features, large λ leads to little smoothing so that details are usually preserved well. On the other hand, for large features, small λ leads to smoothing so that noise is removed considerably. Referring to [3], it can be shown that the minimization problem (1) is related to a constrained optimization problem of the type:

$$\min_u \; TV(u) \;\text{ subject to } \int_{\Omega^\omega} |u-z|^2\; dx \le \sigma^2 \left|\Omega^\omega\right| \;\text{ for all windows } \Omega^\omega \text{ in } \Omega, \qquad (2)$$

where σ^2 is the noise variance, estimated from the image. Compared with the common TV-model, the constraint in the model (2) is confined to each local region Ω^ω instead of the whole image. Considering a restored image u from the TV-model with a relatively small λ, the residual $r = z-u$ will include noise as well as details. Then, the violation of the local constraint in (2) reflects the distribution of details in the image. Roughly speaking, whenever the constraint in (2) is satisfied, it is assumed that in Ω^ω the residual primarily consists of noise; otherwise, significant image details are left in the residual. Therefore, λ needs to be increased in region of constraint violation in order to preserve the details in the reconstruction. This adjustment depends on a robust upper bound for the (local) constraint. For this purpose, the confidence interval technique from statistics [4] is introduced to automatically adjust λ based on the size of the windows Ω^ω. This yields a parameter-free method, i.e., without necessity of manually tuning parameters. Moreover, the minimization problem (1) is solved by a superlinearly convergent algorithm based on Fenchel-duality and inexact semismooth Newton techniques [3, 5]. The concept of spatially variant regularization is illustrated in **Fig. 1** which shows the effect of different regularization parameters for a numerical test image. If the amount of regularization is too small, residual noise remains in the image. With higher regularization, noise is eliminated, but small image features start to disappear. The use of two different regularization parameters for the same image yields a result where noise is eliminated, and small features are preserved.

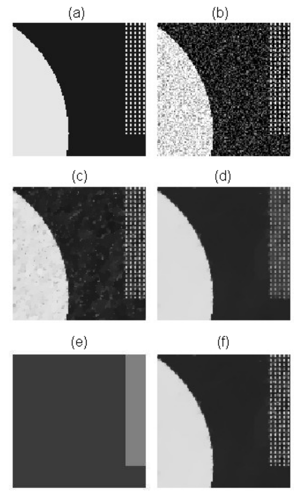

Fig. 1*: A numerical example of spatially variant regularization. (a) A numerical test image. (b) Noisy test image. (c) TV denoising with λ=20. (d) TV denoising with λ=10. (e) λ map: λ=10 (dark region) and λ=20 (bright region). (f) TV denoising with spatially variant λ from subplot (e).*

Methods and Results: Two T_2 weighted scans of the prostate and a diffusion tensor (DTI) data set of the brain were acquired on a clinical 3T system (Siemens TIM Trio, Erlangen, Germany). Written informed consent was obtained prior to the examinations. Sequence parameters of the first T_2w scan were TR = 3300ms, TE = 107ms, matrix size = 320x320 covering a FOV of 170x170mm^2, 25 slices with a slice thickness of 3mm, 3 averages, turbo factor 13. Sequence parameters of the second T_2w scan were TR = 4290ms, TE = 116ms, matrix size = 512x512 covering a FOV of 180x180mm^2, 21 slices with a slice thickness of 3mm, 3 averages, turbo factor 15. DTI data were acquired with a diffusion weighted single shot SE-EPI sequence (TR/TE = 6.7s/95ms, in plane resolution = 1.95x1.95mm2, slice thickness = 2.5mm, 4 averages), with diffusion sensitizing gradients applied in 12 independent directions (b = 1000s/mm²) and an additional reference scan without diffusion. Denoising was performed as a post-processing step using a Matlab implementation of the proposed spatially dependent regularization parameter selection TV method. DTI scans were then processed with FSL [6] including eddy current correction, brain skull extraction, diffusion tensor calculation and visualization.

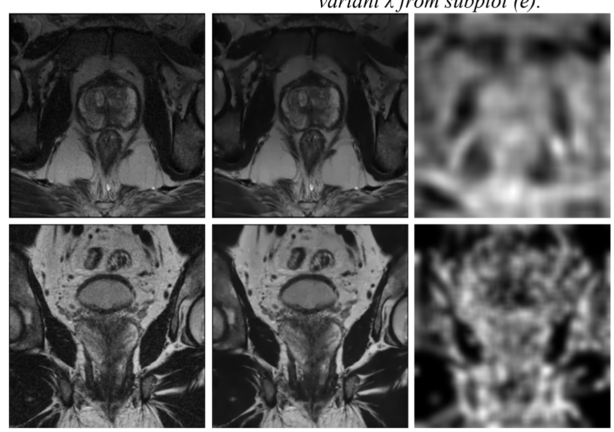

Fig. 2*: Denoising results from a single slice of both datasets. Original Image (left), proposed TV method with spatially dependent regularization (middle), regularization parameter map (right). Bright values in the parameter map correspond to high values of λ, which in turn means that the amount of regularization is small in this region.*

Discussion: The results from this work show that with spatially dependent regularization we obtain excellent image quality and preserve fine details (**Fig. 2**). It can also be seen in the parameter map that the algorithm delivers a good estimate of the details in different regions of the images. These parameter maps also provide additional information about the reconstruction quality for the user, because they highlight the regions where a large amount of filtering was performed. DTI results are shown as fractional anisotropy (FA) maps, which are color-coded according to the main eigenvector of the tensor (Fig. 2). Post-processed fiber tracts are more homogeneous regarding direction and FA value, whereas details of fine fiber tracts are still maintained (**Fig. 3**). This is in line with the morphology underlying and might improve applications such as fibertracking or segmentation of DTI data. From the constraint in (2), we can see that this work is based on the assumption that the noise variance σ^2 is uniform in the whole image. This assumption is not valid in some imaging situations, e.g. when using phased array coils or parallel imaging, which leads to local noise amplifications. However, when the image includes small differences of the noise level, with slightly smaller variance estimation, the results of this work are still acceptable. Even if the images are composed of several pieces with noticeable different noise levels, our algorithm can be implemented similarly for heterogeneous σ^2-values case, i.e., σ^2 in the constraint of (2) is spatially dependent instead of a constant. In this case, information about local noise amplification, e.g. g-factor maps for specific coil geometries, can be included in the algorithm. This will be investigated in future work. With our Matlab implementation, which was not optimized in terms of computational efficiency, the computation times were 55s for a single slice of the first data set (matrix size 320x320), and 260s for the second data set (matrix size 512x512). An important feature of the proposed method is that it is completely free of parameter selections. The regularization parameter is adjusted automatically based on the noise level, and the distribution of the details in the image.

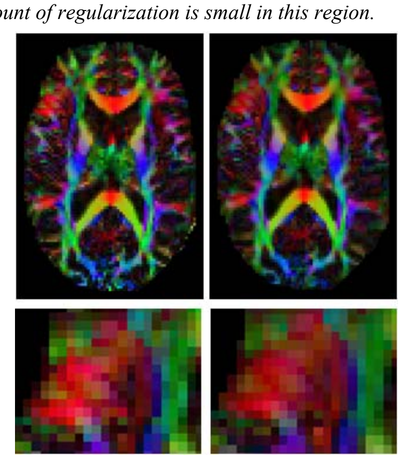

Fig. 3*: FA maps from the original (left), and the denoised (right) DTI data set. Magnified views of a ROI (bottom) demonstrate feature preservation in fine structures.*

References: [1] Rudin et al., Phys. D, 60(1-4) 259-268 (1992), [2] Block et al., MRM 57: 1086-1098 (2007), [3] Dong et al., IFB-report, 22(11/2008), [4] Mood, Introduction to the Theory of Statistics, (1974), [5] Hintermueller et al., SIAM J. Sci. Comput. 28: 1-23 (2006), [6] Smith et al., NeuroImage, 23:208-219 (2004)