# Compressed Sensing Reconstruction in the Presence of a Reference Image

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**Introduction:** Sparsity is an essential condition for compressed sensing (CS). Conventional CS-based MRI method relies on finding a good sparsifying transform in order to produce high quality CS-based reconstructions [1]. If sufficient sparsity cannot be achieved, CS-based reconstructions from reduced samples will typically contain artifacts. In this work, we aim at further improving signal sparsity using a reference image, which is available in various MRI applications, such as dynamic contrast-enhanced imaging and interventional imaging. One straightforward approach is subtracting the reference data from the acquired data to form a "new" data set and the corresponding difference image is reconstructed using a CS-based algorithm [2]. However, the presence of object motion, as is often the case in many applications such as interventional imaging, will degrade the improvement of sparsity. To address this issue, this work presents a compressed sensing reconstruction scheme with a novel motion compensation algorithm.

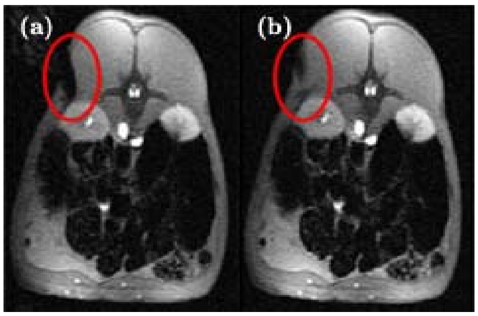


***Fig.1**: Testing frames: (a) reference; (b) target*

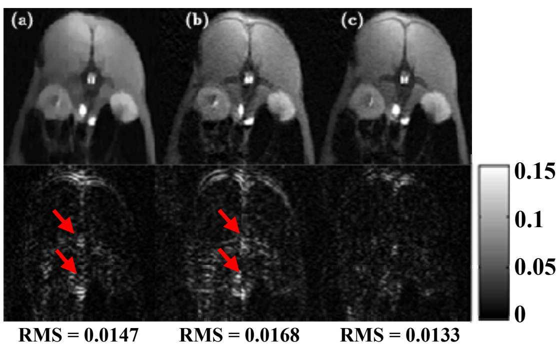


***Fig.2** Reconstruction comparison: first row (a) conventional CS; (b) CS + uncompensated reference; (c) CS + motion compensated reference. Second row: corresponding error maps and RMS errors compared with Fig. 1(b).*

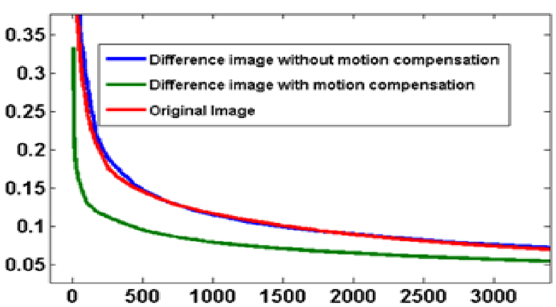


***Fig.3** Signal decay in the finite different domain (zoomed-in). Horizontal: pixel number; vertical: intensities*

**Theory:** The image model we used is: $I_t(\vec{r}) = I_r(T(\vec{r})) + I_d(\vec{r})$, where $I_t(\vec{r})$ is the target image to be reconstructed, $I_r(T(\vec{r}))$ is a deformed reference with a coordinate transform $T$ characterizing the object motion, and $I_d(\vec{r})$ is the difference image. To estimate $T$, we assume a low resolution data set is collected from the target image in addition to a set of randomly sampled higher frequency k-space data. From the low resolution data, a low resolution estimate of $I_t(\vec{r})$ is obtained. We then minimize the metric: $\left\| \nabla \left( \hat{I}_r(T(\vec{r})) - \hat{I}_t(\vec{r}) \right) \right\|_1$ (*), to estimate $T$ based on two low resolution images $\hat{I}_r$ and $\hat{I}_t$. The proposed metric is chosen according to the assumption that when motion compensated, the difference image should be sparser in the finite difference domain. By using the proposed metric, we expect to find a motion compensated reference which promotes the sparsity in the finite difference domain of the difference image. After a proper $T$ is obtained, the difference image is reconstructed using a CS-based reconstruction algorithm as follows:

$$I_d^* = \arg\min_{I_d} \left\{ \left\| FI_d - \left( d_m - \Omega d_r \right) \right\|_2^2 + \lambda TV\left( I_d \right) \right\} \qquad (1)$$

where $F$ is the encoding matrix, $\Omega$ is the sparse sampling operator, $d_m$ is the measured data and $d_r$ is the data generated from the deformed reference. Subsequently, $I_d^*$ is added to the deformed reference to form the overall reconstruction:

$$I_t^* = I_r(T(\vec{r})) + I_d^* \qquad (2).$$

**Methods:** We have evaluated the proposed method for different motion models in different application scenarios, one of which is shown in Figs.1-2. In this left kidney interventional imaging experiment, the series of images experiences a non-rigid deformation during the intervention operation. The data was acquired using a FLASH sequence, with a 4-coil configuration. Sum-of-squares reconstruction was applied to obtain a series of images. They were used to generate the data for simulation. Fig.1 shows two frames reconstructed by sum-of-squares in which the catheter positions are different (marked by red circles). For comparison, Fig.1 (b) is considered as gold standard target image and Fig.1 (a) as the reference. In this particular case, $T$ is modeled as a B-spline based deformation field [3] [4], which is effective in describing a wide range of physiological motions. A variable density sampling pattern with 32 dense low frequency phase encodings and 20 random high frequency phase encodings is used. The acceleration factor is 256/52≈5. The densely sampled low frequency data is used to estimate the low resolution images for estimating $T$ and all the data is used for reconstruction.

**Results:** Figure 2 shows the comparison of reconstruction results. The proposed reconstruction scheme with motion compensated reference outperforms the conventional CS-based reconstruction (less artifact and smaller error) and the reconstruction using reference without compensation is the worst. The red arrows in the figure point out the apparent artifacts which are reduced in the reconstruction using the proposed method. These results are in good agreement with the coefficients decay in the finite difference domain shown in Fig. 3. The difference image obtained from the motion compensated reference gives best compressibility in the finite difference domain.

**Conclusion:** A compressed sensing reconstruction scheme in the presence of a reference image is proposed. Motion between the reference and the target image is compensated. Better reconstruction is achieved with the same acceleration factor compared with conventional compressed sensing reconstruction.

**Reference:** [1] Lustig M *et al, MRM* 58: 1182-1195, 2007. [2] Jim Ji, *et al, ISBI,* 1613-1616, 2008. [3] D. Rueckert et al, *IEEE TMI* 8, 712-721, 1999. [4] Szeliski R *et al, IJCV* 22, 199-218, 1997.

**Acknowledgement** The authors acknowledge Dr. Jeffrey L. Duerk from Case Western Reserve University for providing the intervention experimental data.