Image Reconstruction from Radially Acquired Data using Multipolar Encoding Fields

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Introduction: Recently, it has been shown that conventional imaging techniques are applicable, when strongly nonlinear and non-bijective encoding fields (SEMs) are used for gradient encoding. Only minor design modifications to existing MR sequences are required [1]. In a basic implementation of PatLoc imaging [2] such fields are applied in place of the conventional linearly varying gradient fields. The non-bijectiveness of the SEMs leads to ambiguities, which are resolvable by means of suitable parallel imaging reconstruction techniques [2]. So far, the new PatLoc imaging concept has only been reviewed with respect to Cartesian reconstruction and sampling strategies [1]. However, standard radial acquisition schemes also have proven to be advantageous in many applications. The purpose of this work is to investigate reconstruction strategies for radial acquisition schemes with a focus on multipolar fields (Fig. 1) for gradient encoding.

Theory: The signal equation in PatLoc imaging differs from the standard case of parallel imaging with respect to the phase factor via the substitution $\vec{x} \rightarrow \vec{\varphi}(\vec{x})$ [2]:

$$s_\alpha(\vec{k}) = \int m(\vec{x}) c_\alpha(\vec{x}) e^{i\vec{k}\vec{\varphi}(\vec{x})} d\vec{x}$$

Here, s_α is the signal of rf-coil α with rf-coil sensitivity c_α and m denotes the magnetization. The components $\varphi_j(\vec{x})$ of the multidimensional function $\vec{\varphi}(\vec{x})$ are directly proportional to the sensitivity of the encoding fields. For 2D imaging with multipolar fields having n poles, $\vec{\varphi}$ is most conveniently described in polar coordinates (r,ϑ). It has the simplest representation upon the diagonal variable transformation $(r,\vartheta) \rightarrow (r',\vartheta') = (r^n, n\vartheta)$:

$$\vec{\varphi}(r,\vartheta) = r^n \begin{pmatrix} \sin(n\vartheta) \\ \cos(n\vartheta) \end{pmatrix} = r' \begin{pmatrix} \sin(\vartheta') \\ \cos(\vartheta') \end{pmatrix}.$$

The dependency of $\vec{\varphi}$ on r can be interpreted separately from ϑ: There is a linear acceleration in the azimuthal direction with equidistant n-fold aliasing. In the radial direction a distortion of r^n occurs. The transformation to (r',φ') makes the application of conventional reconstruction techniques possible, although the encoding fields are not linear.

Methods: The projection-slice theorem states that a 2D-Fourier Transform is equivalent to a ray-wise 1D Fourier Transform with subsequent inverse Radon Transform.
Applied to radial imaging this means that there are in principle two basic ways of how to reconstruct a PatLoc encoded image based on radial trajectories:

A) Gridding the signal values onto a Cartesian grid, applying a 2D Discrete Fourier Transform and subsequently Cartesian PatLoc reconstruction as described in [1].
B) Applying a 1D Discrete Fourier Transform ray-wise, then an inverse Radon Transform and subsequently Cartesian PatLoc reconstruction.

These two reconstruction pathways are illustrated in Fig 2. In standard implementations of the inverse Radon Transform the data are finally calculated on a Cartesian grid. However, as motivated in the theory section, for multipolar fields it is favorable to work with polar coordinates or with the transformed (r',ϑ') -coordinates except for visualization of the image, where Cartesian coordinates are preferred. It is straightforward to calculate the image values of the inverse Radon Transform at arbitrary image positions, like on a regular grid in (r',ϑ') -space. It is therefore possible to perform the reconstruction directly in transformed polar coordinates. This results in the alternative reconstruction pathway C depicted in Fig. 2. The reconstruction pipeline of C is shown in more detail in Fig. 3 at the bottom of the page. The steps of the algorithm are accompanied by measurement results. The measurements were performed on a 3T Tim Trio (Siemens, Germany) with a PatLoc-coil insert [4] producing nearly pure quadrupolar fields.

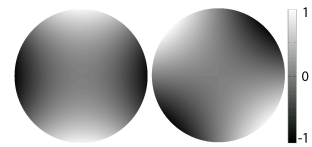

Figure1: Orthogonal multipolar PatLoc-encoding fields of second order for 2D-imaging. Each field defines a different component forming the phase factor of the PatLoc signal equation.

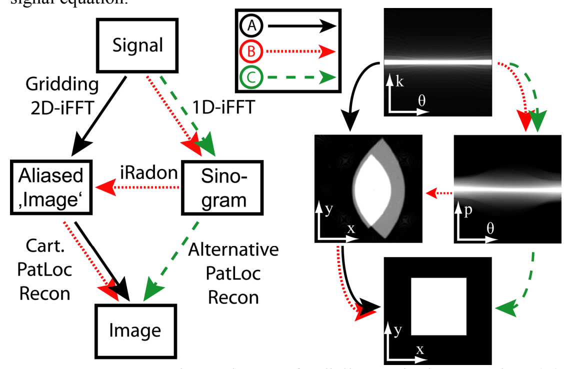

Figure 2: Reconstruction pathways of radially acquired PatLoc data. (A) Gridding of k-space data onto a Cartesian grid, 2D-iFFT and Cartesian PatLoc reconstruction. (B) 1D-iFFT of signal rays, inverse Radon Transform and Cartesian PatLoc reconstruction. (C): Alternative reconstruction using transformed polar coordinates (details in Fig. 4).

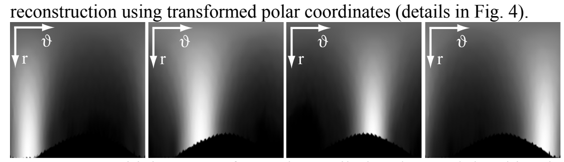

Figure 4: Sensitivity maps of a receiver-coil placed around the object in a circumferential way, depicted in polar coordinates. There is only a small overlap, especially for larger radii resulting in favorable g-factors for the SENSE reconstruction.

Results and Discussion: In Fig. 3 measurement results are shown for the alternative reconstruction pathway C. The image properties of the reconstructed image are similar to those previously found for Cartesian trajectories [5,6]. However, an exact analysis of resolution, SNR and artifacts is beyond the purpose of this contribution. The main advantages of the alternative reconstruction presented here are:

1. The unfolding process can be performed with the standard Cartesian SENSE algorithm [3]. Sensitivity data must be represented in polar coordinates (Fig. 4).
2. Intensity correction has to be performed only in the radial direction (multiplication with r^n)
3. No distortion correction is needed

For other than pure multipolar encoding fields, pathway C is not applicable. However, methods A or B can always be applied to reconstruct images from radially acquired data, also in the context of strongly nonlinear and non-bijective encoding fields. For multipolar encoding fields, the presented reconstruction method C is preferred as it offers several advantages over the standard methods A or B.

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References: [1] Schultz et al., Proc. ISMRM, #786, 2008; [2] Hennig et al., MAGMA 21(1-2):5–14, 2008; [3] Pruessmann et al., MRM 42:952 –62, 1999; [4] Welz et al., Proc. ISMRM, #3073, 2009; [5] Schultz et al., Proc. ISMRM, #762, 2009; [6] Schultz et. al., Proc. ISMRM #563, 2009.

Figure 3: Reconstruction pipeline of pathway C (cp. Fig. 2). In conventional radial imaging, the reconstruction only consists of a 1D-iFFT with subsequent application of the inverse Radon Transform onto a Cartesian grid. This is modified here by first reconstructing on a polar grid. After intensity correction in the radial direction, standard Cartesian SENSE is used for the unfolding process. Finally, the image is visualized by transforming the image back to Cartesian coordinates.