An Analytical Description of Balanced SSFP with Finite RF Excitation

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Introduction. Conceptually, the only flaw in the common steady state free precession (SSFP) signal theory is the assumption of quasi-instantaneous radio-frequency (RF) pulses. Only recently, moderate to severe signal deviations from the Freeman-Hill formulae (1) were observed for non-instantaneous RF pulses, especially for low relaxation time ratios (T_2/T_1) and in the limit of large flip angles (2). In this article, an exact analytical solution for the balanced SSFP (bSSFP) signal will be derived for use with finite RF pulses being valid over the whole range of flip angles (0° < α < 180°), tissue and sequence related imaging parameters.

Theory. For finite RF pulses, it was shown that transverse relaxation processes are overestimated by a fraction of the RF time $T_{RF}^* := \zeta T_{RF}$ ($0 < \zeta < 1$) and, pictorially, excitation was split into a period of a fully longitudinal and two partially transverse alignments, respectively. The action of an α-pulse is thus proposed to be equivalent to three free precession 'FP' periods (with duration $(T_{RF}^* - T_{RF}^*)/2$, T_{RF}^* and $(T_{RF}^* - T_{RF}^*)/2$, respectively) interplayed by two quasi-instantaneous α/2-pulses (Fig. 1) and is given by

$$\mathbf{R}_\alpha := \mathbf{FP}\left[\left(T_{RF}-T_{RF}^*\right)/2\right]\oplus\mathbf{R}_{\alpha/2}\oplus\mathbf{FP}\left[T_{RF}^*\right]\oplus\mathbf{R}_{\alpha/2}\oplus\mathbf{FP}\left[\left(T_{RF}-T_{RF}^*\right)/2\right] \qquad [1]$$

Here, a symmetric RF pulse was assumed; however, the extension to non-symmetric pulses is conceptually straightforward. Within $TR(n)$, the time evolution of the steady state is thus given by a repetitive block of form

$$\left\{\mathbf{FP}\left[\left(T_{RF}-T_{RF}^*\right)/2\right]\oplus\mathbf{R}_{\alpha/2}\oplus\mathbf{FP}\left[T_{RF}^*\right]\oplus\mathbf{R}_{\alpha/2}\oplus\mathbf{FP}\left[\left(T_{RF}-T_{RF}^*\right)/2\right]\oplus\mathbf{FP}\left[TR-T_{RF}\right]\right\}_n \qquad [2]$$

Inspecting expression [3], it appears elegant to merge the FP periods preceding and proceeding the action of the α/2-pulse, but it may be important to note that this leads to a formal reduction of RF pulse, i.e., from T_{RF} to T_{RF}^* (see Fig. 2). As a result, the repetitive unit finally reads

$$\left\{\mathbf{R}_{\alpha/2}\oplus\mathbf{FP}^*\oplus\mathbf{R}_{\alpha/2}\oplus\mathbf{FP}^{**}\right\}_n \qquad [3]$$

where for shorthand notation, an uppercase single asterix is used to denote periods relating to T_{RF}^*, whereas an uppercase double asterix indicates periods of duration $TR - T_{RF}^*$. From this, using the common matrix formalism, an eigenvector equation can be formulated and in the limit of $TR \ll T_{1,2}$, the on-resonance bSSFP solution is of form

$$M_{xy}^+ \propto \frac{\sin\alpha + \lambda^*\left(2\sin(\alpha/2)-\sin\alpha\right)}{\left(1+\lambda^*\right)+\left(1-\lambda^*\right)\left(\Lambda-\cos\alpha(\Lambda-1)\right)} \qquad [4]$$

where $\lambda := T_{RF} / TR$ and $\lambda^* := T_{RF}^* / TR$ are the fractional RF durations for the finite and reduced finite RF pulse, respectively and $\Lambda := T_1/T_2$ refers to the relaxation time ratio.

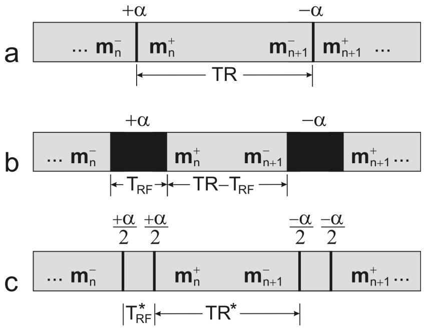

Fig. 1: Balanced SSF RF train with instantaneous α-pulses (a) and finite α-pulses (b) being formally equivalent to two α/2-pulses separated by a period T_{RF}^* of free precession.

Materials & Methods. Numerical integration of the Bloch equation was performed using a standard solver (based on an explicit Runge-Kutta formula) for non-stiff ordinary differential equations. Balanced SSFP signal with finite RF pulses exactly followed the sequence protocol and was simulated as stated in any detail elsewhere (2). For imaging, a 1.5 T system (Siemens Espree) was used and 3D acquisitions (2x2x2mm resolution) with non-slice selective hard pulses were performed. The TR was fixed to 10ms allowing T_{RF} = 600–8400μs for flip angles α = 0°–180°. Experiments were performed on aqueous probes only (to circumvent magnetization transfer issues) with T_2/T_1=120ms/140ms ~ 1 and (ii) T_2/T_1=150ms/1100ms ≈ 0.14 << 1.

Results & Discussion. Estimates for the parameter ζ are derived from a series of finite difference simulations of the bSSFP signal with finite RF pulses (not shown). In general, ζ depends only on the parameters Λ and λ, since the signal of bSSFP is a weighted combination of T_2 and T_1 and shows only marginal variation with TR (for $TR \ll T_2$). Analysis of ζ as a function of λ and Λ^{-1} suggests (not shown)

$$\zeta \approx \zeta_0 - \zeta_2\cdot\left(\frac{\alpha}{\pi}\right)^2, \text{ with } \zeta_0 = \frac{2}{3},\ \zeta_2 = \frac{1}{30}\left(1+2\lambda\left(1-\Lambda^{-1}\right)\right) \qquad [5]$$

The accuracy of bSSFP signal models is shown in Fig. 2 in the limit of $T_{RF}/TR \rightarrow 0$ and $T_{RF}/TR \rightarrow 1$: The Freeman-Hill formulae is accurate, as expected, for instantaneous RF pulses but a considerable signal underestimation is observed with finite RF pulses. The recently proposed T_2-substitution scheme is accurate for α < 90°, but fails at higher flip angles, whereas Eq. [4] is accurate over the whole range of flip angles. A general framework for the description of bSSFP signal formation with finite RF pulses was introduced. It was shown that finite α-pulses decompose into two α/2-pulses interleaved by a FP period. The duration of FP period is determined by the factor ζ which expresses the mean fraction of the RF time the steady state magnetization is pointing along the longitudinal direction. Excellent agreement between finite RF theory and experiments were observed over the whole range of flip angles and relaxation times.

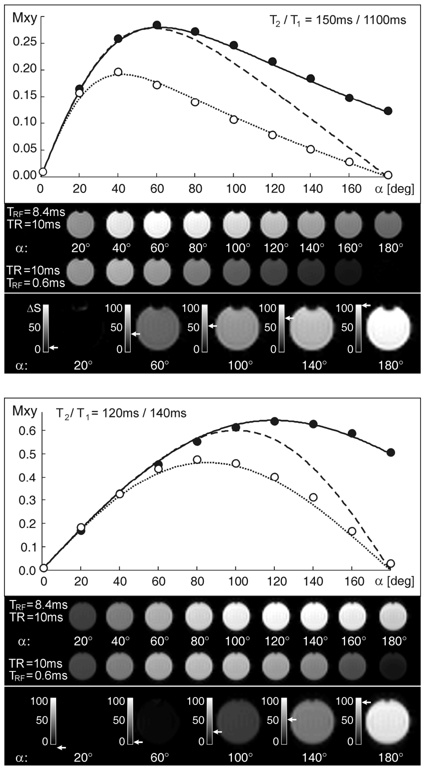

Fig. 2: Measured bSSFP transverse magnetization (M_{xy}) as a function of α in the limit of both almost continuous (●, S1) and instantaneous RF excitation (○, S2) with corresponding observed relative signal deviation Δs = (S1 – S2) / S1. The dotted line indicates M_{xy} from common bSSFP signal theory (Freeman-Hill), the dashed line represents T2-corrected intensities as described in (2) and the solid line corresponds to the analytical signal equation with finite RF pulses according to Eq. [4]

Conclusion. Typically *TR* is short with SSFP and finite RF effects can be quite significant even for moderate RF pulse durations. Thus care should be taken when interpreting SSFP signal based on the common Freeman-Hill formulae since only recently it was realized that besides finite RF pulses also magnetization transfer effects may induce a significant modulation in the steady state amplitude.

References. Freeman, R, Hill H. J. Magn. Reson 1971; 4:366-383. Bieri, O, Scheffler, K. MRM 2009; 62(5):1232-1241.