General Formulas for Optimizing Two-Point Saturation-Recovery Measurements

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Introduction. Measuring the longitudinal relaxation rate R_1 is common in quantitative MRI. The saturation–recovery (SR) method is a popular technique for the R_1 measurement, in which the longitudinal magnetization is set to zero and then measured after a delay time τ (referred to as the *recovery time*). The value of R_1 is determined from separate measurements of at least two different recovery times. An interesting, long-standing question in clinical applications is, given a limited amount of time, how the recovery times should be allocated to minimize the uncertainty of the measurement. An analytical approach [1] would not be practical because of the increased mathematical complexity when the total imaging time is subject to a constraint. Recently, a Monte Carlo computational study [2] was performed to answer this question and the results were demonstrated successfully with 3D, high resolution, whole brain R_1 mapping. It was found that, given the same total imaging time, acquiring two recovery times (*i.e.*, two points) might be preferable to three points because the precision improvement is only marginal and may not outweigh the problems introduced by the third recovery time (*e.g.*, inter-scan subject motion, rf exposure, limitation of minimal pulse spacing, data storage). The simulation was performed for a few special cases needed for mapping the cerebrospinal fluid, which has long T_1 ($T_1 \sim 4$ s). To facilitate quantitative MRI in clinical applications, a systematic Monte Carlo computation for the two-point SR is carried out in this work to derive general formulas covering a range also useful for R_1-mapping the tissue.

Methods. The SR equation is given by $M(\tau) = M_{eq}\,[1 - \exp(-\tau R_1)]$, where $M(\tau)$ and M_{eq} represent the magnetization at recovery time τ and at thermal equilibrium, respectively. Given two data $M(\tau_1)$ and $M(\tau_2)$, the SR equation is solved computationally to obtain R_1 by using the Newton's bisection algorithm. In this work, T_1 represents the true longitudinal relaxation time of the specimen and is used as the unit of time; R_1 is the longitudinal relaxation rate derived from data and is expressed in units of $1/T_1$. The time constraint on the total imaging time is specified in terms of the sum of the recovery times (SRT) $T_\Sigma = \tau_1 + \tau_2$. §***Computer Simulation.***—The distribution of R_1 values obtained by solving the SR equation was simulated by using a large number of computer-synthesized data. Fourteen signal-to-noise ratios (SNR) ρ in [50, 64000] were considered. The data were synthesized as the SR equation plus Monte Carlo noise [zero-mean, normally-distributed with the standard deviation (SD) determined by the SNR]. For each SNR, an R_1 distribution (consisted of 65536 trials) was established for each vertex on a two-dimensional Cartesian grid of recovery-time spacing $0.05T_1$; because of the symmetry, only vertexes of $\tau_1 < \tau_2$ are considered. §***Optimal Recovery Times.***—For each desired SRT, the vertex of the smallest SD was searched and then its SD together with those of the two nearest neighbors on each side (if available) were included to fit a second order polynomial of τ_1 for smoothing. Then the ideal set of the recovery times (τ_1, $\tau_2 = T_\Sigma - \tau_1$) was determined as the one that minimizes the polynomial. The search was repeated for the SRT in the interval $0.05T_1 \le T_\Sigma \le 15T_1$ with grid spacing of $0.25T_1$ to establish the optimal recovery times as functions of the SRT.

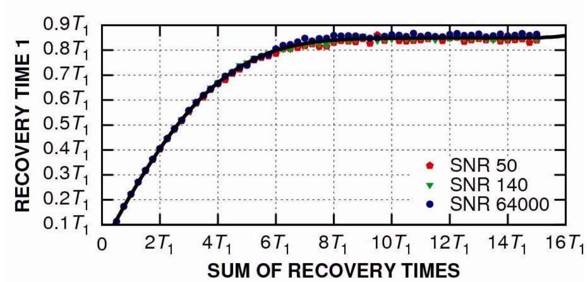

Figure 1 Optimal recovery time as a function of sum of recovery times.

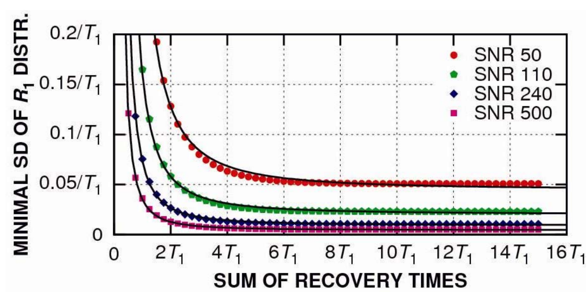

Figure 2 Minimal standard deviation of the computer-simulated R_1 distribution.

Results and Discussion. §***Optimal Recovery Times vs. SRT.***—Figure 1 shows the optimal τ_1 for a few sample SNR levels and the curve of the least-squares fitting to all SNR levels as a whole: $\tau_1(T_\Sigma) = 1.566\times10^{-2} + 2.086\times10^{-1}T_\Sigma + 1.182\times10^{-3}T_\Sigma^2 - 5.257\times10^{-3}T_\Sigma^3 + 6.585\times10^{-4}T_\Sigma^4 - 3.318\times10^{-5}T_\Sigma^5 + 6.182\times10^{-7}T_\Sigma^6$, where $0.05T_1 \le T_\Sigma \le 15T_1$. This formula also reproduces the three special cases in Ref. [2]. In the SNR range considered, the minimal SD occurs at the same set of the recovery times despite the SNR. Beyond SRT of $\sim 8T_1$, lengthening the SRT merely means adding time to the longer recovery time (τ_2). §***Precision of the SR Measurement.***—Figure 2 shows samples of the SD of the R_1 distribution when the recovery time setting is optimal, which can be described roughly by $\sigma_{min}(T_\Sigma, \rho) = -1.305\times10^{-5} - 2.386\times10^{-7}T_\Sigma + (2.206 + 1.399T_\Sigma^{-1} + 13.915T_\Sigma^{-2})\rho^{-1}$, for $0.05T_1 \le T_\Sigma \le 15T_1$ and $50 \le \rho \le 64000$. This formula can help estimate the expected error when averaging multiple R_1 measurements or determine the number of measurements required to reach the desired precision; an example is given in Ref. [2]. From Figs. 1 and 2, increasing the SRT to improve precision is effective only up to a limit (*e.g.*, SRT $\sim 8T_1$ for SNR of 50) and the improvement is more dramatic when the SRT is short. These general formulas are expected to be useful in designing SR experiments.

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References: [1] See, for example, GH Weiss *et al.*, J Magn Reson **37**, 369 (1980); ED Becker *et al.*, *ibid.* **37**, 381 (1980); H Hanssum and H Rüterjans, *ibid.* **39**, 65 (1980); SJ Doran *et al.*, *ibid.* **100**, 101 (1992); RJ Kurland, Magn Reson Med **2**, 136 (1985). [2] J-J Hsu, GH Glover, and G Zaharchuk, Magn Reson Med **62**, 1202 (2009).