Spiral MRI Trajectory Design with Frequency Constraint

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Introduction Spiral *k*-space trajectories are used in MRI due to their high efficiency in exploiting the gradient capacities and their low sensitivity to flow [1]. For a practical implementation of a spiral design an efficient calculation of the parameterisation is required while taking into account the limitations of the gradient system, namely maximum amplitude G and slew rate S. For this purpose King proposed to split the parameterisation into two domains, the first one constrained by S and the second one by G [2]. Approximate solutions of the associated differential equations were presented by Heid [3] and subsequently improved by Glover [4] and Cline [5]. However, despite including the restrictions for G and S, the actually realised spiral trajectory can deviate from the expected one. This happens if the gradient waveform contains frequencies beyond the bandwidth limit of the gradient system. In this work the latter is introduced as an additional constraint for gradient waveform design, leading to an improved fidelity of spiral trajectories.

Theory Eqn. (1) summarises the parameterisation of the Archimedean spiral by Cline [5] given as a continuous rotation angle $\varphi(t)$ with gyromagnetic ratio γ, field-of-view D, number of interleaves M, and transition point τ between slew rate- and amplitude-limited domains. As a time-resolved measure of the frequency of a gradient waveform the instantaneous frequency [6, 7] is employed given by the time-derivative of the angle of the complex gradient amplitude $g(t)$ according to Eqn. (2). This approach is used to limit the frequencies of a gradient waveform to a maximum value F. In the Cline design this leads to an upper limit for β of $\beta_F = (7.825\,F)^2$. However, lowering β reduces the slew rate which can result in a greatly increased spiral duration T. Therefore, a three-domain parameterisation is proposed with an initial frequency-limited domain, followed by two domains adapted from the Heid solutions [3] (Eqn. (3)). The value of B in the modified constant angular velocity spiral φ_F was found by numerical optimisation. In order to include also the spectral width associated with the duration of the first domain $\tau 1$, an additional condition is introduced (Eqn. (4)) that is fulfilled in an iterative procedure.

$$\varphi_S(t) = \beta t^2 \left(2 + 2\left(\frac{2\beta}{3}\right)^{1/6} t^{1/3} + \left(\frac{2\beta}{3}\right)^{2/3} t^{4/3}\right)^{-1} \quad (1)$$

$$\varphi_G(t) = \sqrt{\varphi_\tau^2 + 2\kappa(t - t_\tau)},\ \beta = \frac{\gamma G D}{M},\ \kappa = \frac{\gamma S D}{M}$$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\angle g(t), \quad g(t) = g_x(t) + i\,g_y(t) \quad (2)$$

$$\varphi_F(t) = \frac{(At)^2}{B + At},\ A = 2\pi F,\ B = 3.35$$

$$\varphi_S(t) = \left(\varphi_{\tau 1}^{3/2} + \frac{3}{2}\sqrt{\beta}\,(t - t_{\tau 1})\right)^{2/3} \quad (3)$$

$$\varphi_G(t) = \sqrt{\varphi_{\tau 2}^2 + 2\kappa(t - t_{\tau 2})}$$

$$\frac{A}{2\pi} + \frac{1}{\tau 1} \le F \quad (4)$$

Methods Simulations were based on a Fourier analysis of the gradient waveforms. Synthetic *k*-space data was generated by discrete FT of an object, and images were reconstructed using standard gridding. Real data was acquired on a Bruker BioSpec at 7 T and reconstructed using measured trajectories.

Results Fig. 1 shows simulation results for three different designs using G = 40 mT/m, S = 600 T/m/s, and D = 10 cm. A single-shot protocol with matrix size 32 was chosen as the investigated effects are most prominent at the spiral start. The Cline design without frequency limitation (top) exhibits the two domains where gradient and slew rate amplitudes are normalised to their limits while the maximum frequency of 16 kHz is scaled to 1. Also the Fourier analysis shows high frequencies which are damped by the filter representing the frequency response of the gradient system. Due to the damping the actual gradient is reduced (see design plot), the trajectory has a reduced density in the centre, and the reconstructed image is corrupted. In the second design based on Cline (middle) the application of the frequency constraint with F = 5 kHz limits f (plotted normalised with F), which can also be observed in the Fourier analysis. Only negligible gradient damping occurs, resulting in an improved trajectory and a clean image. However, the reduced slew rate leads to a considerably increased duration T. In the 3-domain design (bottom) F was set to 3 kHz according to the plateau of the filter function. The three domains can be noticed in the design plot with the frequency running close to the limit throughout the first domain and decreasing afterwards. Correspondingly, the intensity drops at the limit in the spectrum. The trajectory is realised as desired, providing the same image quality as before at an only moderately increased T compared to the original design. With the same set of designs, experiments were performed with G = 134 mT/m, S = 6130 T/m/s, and D = 7 cm, resulting in T = 2.2 ms ($F = \infty$), 5.8 ms (F = 15 kHz), and 2.6 ms (F = 10 kHz). Without frequency limitation similar artefacts as in the simulation occur, which are removed for the improved designs (Fig. 2). The advantage of the shorter acquisition with the 3-domain design becomes obvious by the absence of the off-resonance blurring due to a field distortion close to the imaged slice.

Fig. 1

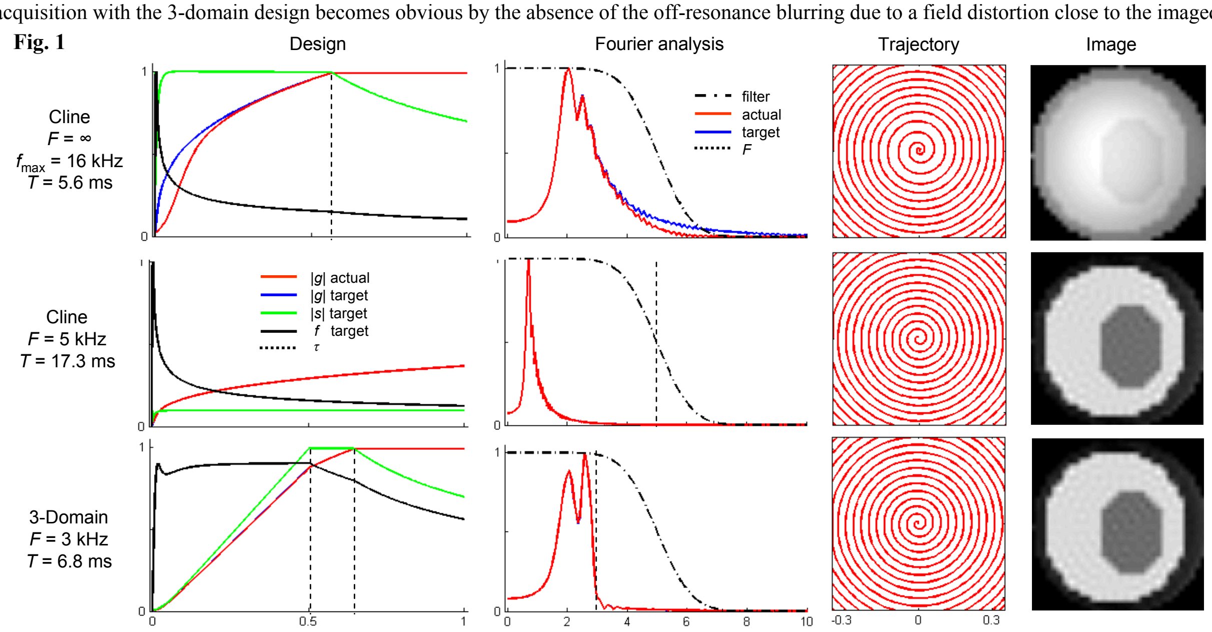
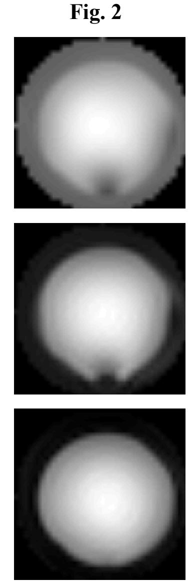


Fig. 2



Conclusion An improved, 3-domain design for Archimedean spiral trajectories has been proposed, utilising the instantaneous frequency for taking into account frequency limitations of the gradient system. The new layout enables creating trajectories with high fidelity and efficiency, leading to improved spiral image quality.

References [1] Nishimura DG, MRM 33 (1995) 549. [2] King KF, MRM 34 (1995) 156. [3] Heid O, ISMRM 1996,114. [4] Glover GH, MRM 42 (1999) 412. [5] Cline HE, MRM 4 (2001) 1130. [6] Rihaczek AW, IEEE Trans. Inform. Theory, 14 (1968) 369. [7] Boashash B, Proc. IEEE 80 (1992) 520.