A General Trajectory Tester

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Introduction While the vast majority of clinical MRI sequences utilize Cartesian sampling strategies, in a growing number of applications non-Cartesian schemes are being employed for a variety of technical reasons. For example, our own motivation is diffusion tensor imaging (DTI) wherein we use spiral trajectories [1], in part because they facilitate efficient self-navigation schemes for artifact mitigation. Non-Cartesian sampling schemes require some method of non-uniform Fourier reconstruction in order to produce an image. Moreover, although most non-Cartesian methods to date have been 2D, there is growing interest in true 3D trajectories. Aside from the theoretical issue of how to design 3D trajectories, there is a significant practical problem of how to practically evaluate such trajectories, which requires two essential elements: 1) A precisely known image (a "gold standard") by which to compare the relative performance of different trajectories, and 2) A general reconstruction method for arbitrary trajectories (and interleaves). A major roadblock to studying the trajectory problem is the lack of a platform within which it is possible to explore the multitude of issues in a practical manner. To address this issue, we have developed a software platform called the General Trajectory Tester (GTT) that allows users to input arbitrary 3D k-space trajectories, in an arbitrary number of interleaves, which are then used to sample and reconstruct the well known 3D analytical Shepp-Logan phantom [2]. Reconstructions can then be quantitatively compared with a true image. At the heart of the GTT is our extension to 3D of the 2D gridded reconstruction method of Inati and Greengard [3] used in our previous work [1], which allows for the Fourier reconstruction of arbitrarily spaced data. The GTT can also simulate diffusion weighting, including arbitrary diffusion angular encoding schemes for DTI, multiple b-values, eddy current and motion induced artifacts and self-navigation, and so is a natural platform to test efficient DTI acquisition and self-navigation schemes.

Coding Framework GTT is written in standard ansi-c, except reconstruction routines, which are written in fortran. Output files (including reconstructed images and k-space data) are written in standard AFNI format (http://afni.nimh.nih.gov/afni/) for easy viewing. From the user's perspective, the use of the GTT is very simple: one chooses the dimension of the problem (2D or 3D), the trajectory (e.g., Cartesian, spiral, 3D, etc.), the imaging parameters (e.g., field of view, matrix size, etc.), the system parameters (e.g. gradient strength, slew rate, etc.) and the phantom (shell or Shepp-Logan), and the GTT reconstructs the complex image. User-defined trajectories can be input with the addition of a single user-supplied c-routine. Options for motion and eddy current artifacts that vary with interleave are also available to simulate DTI related artifacts.

Reconstruction Reconstruction of data acquired on an arbitrary trajectory requires the inversion of data sampled on a non-cartesian grid with a non-uniform density in k-space. As in our previous work [1] we employ the numerical method developed by Inati and Greengard [3] to reconstruct the effective spin density ρ from a finite number of data s, written in compact form [3] as $s = H\rho + \eta$ where η is the noise. We use least squares to find the best estimate $r(\boldsymbol{x})$ of the spin density $r(\boldsymbol{x}) = Ps$ where P is the projection operator that maps the signal onto the space spanned by the operator H that, in turn, maps the spin density (i.e., image space) to the signal (i.e., k) space. P is also called the *pseudo-inverse* of H and is given by $P = H^{\dagger}(HH^{\dagger})^{-1}$ where † denotes the Hermitian conjugate. The complexity of calculating $r(\boldsymbol{x})$ hinges on the invertibility of the matrix $\boldsymbol{B} = HH^{\dagger}$ in P, which is dependent upon the eigenstructure of $\boldsymbol{B}$, where $B_{ij} = \mathrm{sinc}(\boldsymbol{k}_i - \boldsymbol{k}_j)$ and $\mathrm{sinc}(k) = \sin(\pi k)/(\pi k)$. In 3D, $\mathrm{sinc}(\boldsymbol{k}) = \mathrm{sinc}(k_x)\mathrm{sinc}(k_y)\mathrm{sinc}(k_z)$ so that $B_{ij} = \mathrm{sinc}(\Delta k_x)\mathrm{sinc}(\Delta k_y)\mathrm{sinc}(\Delta k_z)$, where $\Delta k = k_i - k_j$. In non-Cartesian schemes, nearly coincident points can produce very small eigenvalues which make $\boldsymbol{B}$ poorly conditioned and its invertibility problematic. However, it can be shown [3] that the pseudo-inverse solution is approximately equal to the weighted Fourier reconstruction $r = \sum_m [w_m s_m \exp(-2\pi i \boldsymbol{k}_m \cdot \boldsymbol{x})]$ where the weights w_m are given by $w_m = [\sum_n \mathrm{sinc}^2(k_m - k_n)]^{-1}$. The direct computation of r, while straightforward, takes $O(n^2)$ and so can be prohibitively expensive for long trajectories, since each point must be summed over all other points. However, it can be calculated in $O(n \log n)$ using the fast sinc^2 transform [4]. For the GTT, we have extended the 2D fast sinc^2 transform (in [4]) to 3 dimensions. The image can then be reconstructed using fast non-uniform Fourier transform [5]. Furthermore, it can be shown that the approximation to the pseudo-inverse is given by the weighted Fourier reconstruction, which can be improved upon via a simple iterative scheme [3].

Results and Conclusion In Figure 1 is shown an example of a complicated 3D trajectory, given as $x(t) = r\cos(2ib)\sin(jb)$, $y(t) = r\sin(2ib)\sin(jb)$, $z(t) = \cos(2jb)$, using 64x64 interleaves{i, j} and with 128 points each in r, along with the resulting reconstruction of the Shepp-Logan phantom. This demonstrates that the GTT reconstructs images of a known analytical phantom for any given 3D trajectory and an arbitrary number of interleaves using a general method for reconstruction of arbitrarily spaced Fourier data. The program allows the easy testing of trajectories, and is also able to simulate diffusion effects in order to test methods for artifact correction by self-navigation.

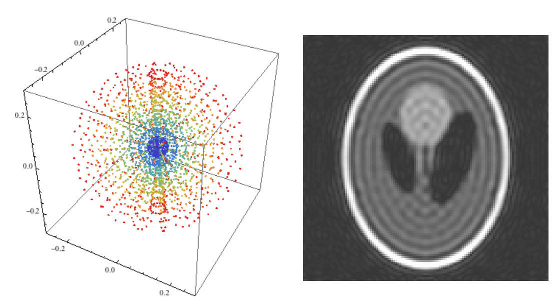

Figure 1. 3D trajectory and resulting reconstruction of the Shepp-Logan phantom.

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