Spatial RF Pulse Design in Local Rotating Frame

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Introduction

Formulation of spatial RF pulse design in a gradient-dependent rotating frame is introduced. This work was motivated by the need for a fast RF design algorithm for use in a high-field parallel transmit system where dielectric effect creates transmit field distribution with significant patient-dependent variability. The new reference frame, called a "local rotating frame", transforms away any (known) longitudinal magnetic fields present in the problem, and thereby greatly simplifies analytical and numerical calculations of spatial RF pulse response in a single-coil or a parallel transmit system. We demonstrate the application of the new method in analytical and numerical solutions of the Bloch equations in the spinor domain.

Theory and Methods

Conventionally, spin dynamics in MRI is formulated in a frame rotating at a fixed Larmor frequency determined by the static field. We define a local rotating frame as a frame which has an additional transverse phase $-\phi(\boldsymbol{r}, t)$ with respect to the conventional frame, where $\phi(\boldsymbol{r}, t) = \gamma\int_0^t (\delta B_0(\boldsymbol{r}) + \boldsymbol{G}(t')\cdot\boldsymbol{r})dt'$. Here $\delta B_0(\boldsymbol{r})$ and $\boldsymbol{G}(t)$ are off-resonance field and applied gradient field, respectively. In such a frame, the transverse magnetization and the complex RF field appear as $M_{xy}(\boldsymbol{r}, t) = M_{xy,c}(\boldsymbol{r}, t)\, e^{\,i\phi(\boldsymbol{r},\, t)}$, and $B(\boldsymbol{r}, t) = B_c(\boldsymbol{r}, t)\, e^{\,i\phi(\boldsymbol{r},\, t)}$, where we used subscript '*c*' to denote quantities in the conventional frame. Additionally, the Cayley-Klein parameters defining three-dimensional rotation transform as $\alpha(\boldsymbol{r}, t) = \alpha_c(\boldsymbol{r}, t)\, e^{-i\phi(\boldsymbol{r},\, t)/2}$, $\beta(\boldsymbol{r}, t) = \beta_c(\boldsymbol{r}, t)\, e^{-i\phi(\boldsymbol{r},\, t)/2}$. The Bloch equations in the new variables lack any longitudinal field, and are considerably simpler. In the spinor domain, the equations in the absence of relaxation are:

$$d\alpha^*/dt = (i\gamma/2)\, B(\boldsymbol{r}, t)\, \beta(t), \quad d\beta/dt = (i\gamma/2)\, B^*(\boldsymbol{r}, t)\, \alpha^*(t) \tag{1}$$

with the initial condition of $\alpha(0) = 1$, $\beta(0) = 0$. Analytical integration of Eqs. (1) from $t = 0$ to T can be done in a series expansion, $\alpha^*(T) = \alpha^{*(0)}(T) + \alpha^{*(2)}(T) + ...$, $\beta(T) = \beta^{(1)}(T) + \beta^{(3)}(T) + ...$ where successive terms are smaller than the preceding term by order of $(\theta_t/2)^2$, where θ_t is the tipping angle in radian. The Linear Class Large Tip Angle pulse theory [1] was formulated based on the first two terms in the expansion. Remarkably, the next term $\alpha^{*(2)}(T)$ can be calculated in a closed form for a one-dimensional selective RF pulse. This term determines the residual dephasing following slice-selective excitation. For a rectangular slice profile the result is,

$$\frac{d\phi_{res}}{dz} \equiv -\frac{1}{2\pi z_{FW}} \cdot \frac{\theta_t^2}{1-\theta_t^2/8} \tag{2}$$

where z_{FW} is the slice thickness, and $d\phi_{res}/dz$ is the phase winding along the slice direction after a usual half-area gradient rewinder is applied. The new frame transformation is particularly beneficial when multiple RF fields are simulated for pre-determined gradient fields [2]. In such a case, the RF simulation can follow the workflow depicted in Fig. 1. Since the gradient field is typically much larger than the rf field for the majority of the voxels, analytical transformation to separate gradient fields away from Bloch simulation provides substantial improvement in the robustness of the numerical integration, allowing a larger step size to be taken. We tested the efficiency of this method for a simulated 8-coil parallel transmit system with spiral gradient for reduced-FOV excitation.

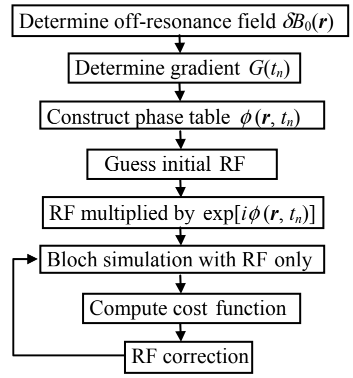

Figure 1. Iterative RF design workflow using local rotating frame transform

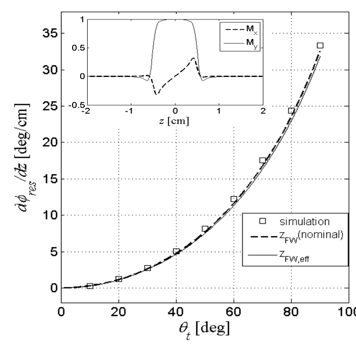

Figure 2. Phase winding from Eq. (2) (solid and dashed lines) compared with Bloch simulation (square). Inset is the slice profile for $\theta_t = 90°$.

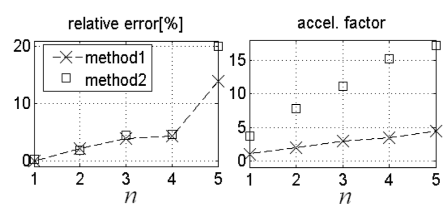

Figure 3. Bloch simulation performance as a function of the step size, $dt = n \times 4$ [μs]

Results and Discussion

Figure 2 shows residual phase slope from Eq. (2) compared with that obtained from linear fitting to the simulated magnetization profile (inset). The RF pulse used was a Hamming-windowed sinc function with time-bandwidth product of 12. Eq. (2) accurately predicts the residual phase, capturing initial quadratic dependence of the phase slope on the tip angle. Figure 3 shows the speed gain (right) and relative error (left) of Bloch simulations in local rotating frame as a function of the integration step size. The simulation was performed on eight independent RF waveforms calculated by linear method [3] based on a 6-turn spiral *k*-trajectory for 90° excitation of a rectangular FOV. Compared are simulations performed by step-by-step rotation at 4*n*-microsecond interval in conventional frame (method 1), and by linear advancement of each time step (Euler's method) in local rotating frame (method 2). Method 1 was considered exact for $n = 1$, and the error was calculated in terms of maximum tip angle deviation referenced to this case. Despite the simplicity of Euler's method integration, Method 2 nearly matches the accuracy of Method 1 for $n < 5$, with consistent speed advantage of about factor of 3.

Conclusion

We have described a theoretical framework for spatial RF pulse design using local rotating frame transformation. The method simplifies analytical and numerical calculations of the RF pulse response, and is straightforward to implement in existing RF design methods. New theoretical insights derived from this method in spatial RF pulse design are detailed in upcoming publication [4]. We believe that the Bloch simulation method outlined here could streamline numerical calculation workflow in iterative RF optimization, with substantial gain in calculation speed.

References

[1] Pauly J et al, J Magn Reson 1989;82:571 [2] Xu D et al, MRM 2008;59:547 [3] Xu D et al, MRM 2007;58:326 [4] Lee SK, submitted to MRM.